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RESEARCH ARTICLE

ELIABILITY CALCULATION OF BAR SYSTEMS UNDER FATIGUE FAILURE

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Abstract. *The article is devoted to the development of methods for assessing the reliability of elastic systems with random parameters of structural elements based on static models. At present, probabilistic methods are increasingly replacing traditional approaches based on allowable stresses and limit states, becoming the foundation of modern building codes and standards. Within this framework, structural failure is considered a random event, while reliability and stability are defined by the probability of failure-free operation. For systems subjected to time-invariant loads and possessing constant material properties, static reliability models provide an effective analytical tool. Despite advances in construction science and engineering, the assessment of structural reliability under random actions remains a highly relevant issue due to increasingly complex operating conditions and growing safety requirements for engineering structures. The scientific novelty of the study lies in the fact that approximate analytical relationships have been obtained for the first time for determining the reliability and service life of elastic structural systems, taking into account the random nature of both variable stress amplitudes and the fatigue limit of materials. A new engineering methodology for the probabilistic analysis and design of multielement mechanical systems has been proposed, incorporating the effect of dependent failures of structural elements. Furthermore, analytical relationships between static reliability models and multifactor random parameters of structures have been established for the first time. The developed methods and analytical relationships have been transformed into engineering algorithms, providing a practical basis for the reliable design of complex load-bearing systems.*

Keywords: *reliability; elastic systems; stress state; tensile strength; durability.*

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ҒЫЛЫМИ МАҚАЛА

ҚАЗУ БҰЗЫЛУЫ БОЙЫНША СТЕРЖЕНДІК ЖҮЙЕЛЕРДІҢ СЕНІМДІЛІГІН ЕСЕПТЕУ

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Андатпа. Мақала статикалық үлгілеулерге негізделген құрылыс конструкцияларының стержендік элементтері мен машина бөлшектерінің кездейсоқ параметрлері бар серпімді жүйелердің сенімділігін есептеу әдістерін дамытып жетілдіруге арналған. Қазіргі уақытта яғни жаңа ықтималдық тәсілдер заманауи құрылыс нормаларының негізіне айнала отырып шектік кернеулер мен шекті рұқсат етілген күйлер мен жағдайлар бойынша дәстүрлі есептеулерді белсенді түрде ығыстыруда. Осы шешімнің аясында құрылым жүйесінің істен шығуы немесе бұзылуы кездейсоқ оқиға ретінде қарастырылады, ал сенімділік бойынша жұмыс істеу ықтималдығы ретінде қарастырылады. Уақыт бойынша өзгермейтін жүктемелері мен қасиеттері бар жүйелер үшін статикалық үлгілеулер тиімді талдау құралы болып табылады. Техниканың жетілдірілуіне қарамастан, кездейсоқ жүктемелер әсерінен конструкциялардың сенімділігін бағалау мәселесі нысандарды пайдалану жағдайларының күрделенуіне және жауапкершіліктің өсуіне байланысты өте өзекті болып қала береді. Зерттеудің ғылыми жаңалығы келесідей болады: Алғашқы болып құрылымдардың серпімді жүйелерінің сенімділігі мен ұзақмерзімділігін анықтайтын аналитикалық тәуелділіктер алынды, сонымен бірге ол құрылыс материалдарының айнымалы кернеу амплитудасын көрсетеді және төзімділік шегінің кездейсоқ сипатын ескереді. Тәуелсіз жағдайлары классикалық схемалардан құралып ерекшеленетін элементтердің тәуелді істен шығуының әсерін ескере отырып, көп элементті құрылымның механикалық жүйелерін ықтималдық есептеу мен жобалаудың жаңа инженерлік әдістемесі ұсынылды және келтірілді. Автордың бұрын жариялаған базалық түрдегі ережелерден айырмашылығы, бұл жұмыста практикалық жобалау есептері үшін қажуды болдырмау теориясы жалпыланған және бейімделіп келтірілген, сонымен қатар статикалық сенімділік үлгілері мен құрылымдардың көп факторлы кездейсоқ параметрлері арасында айқын қатынастармен келтірілу алғаш рет анықталған. Мақалада келтірілген әдістер мен формулалар инженерлік алгоритмдерге жеткізілген, бұл күрделі жүккөтергіш жүйелерді сенімді жобалау үшін практикалық негіз құрайды.

Түйінді сөздер: сенімділік; серпімді жүйелер; кернеуленген күй; төзімділік шегі; ұзақ мерзімділік.

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НАУЧНАЯ СТАТЬЯ

РАСЧЕТ НАДЕЖНОСТИ СТЕРЖНЕВЫХ СИСТЕМ ПО УСТАЛОСТНОМУ РАЗРУШЕНИЮ

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Аннотация. *Статья посвящена развитию методов расчета надежности упругих систем со случайными параметрами элементов строительных конструкций на основе статических моделей. В настоящее время вероятностные методы и подходы активно вытесняют традиционные расчеты по допускаемым напряжениям и предельно - допустимым состояниям, становясь основой современных строительных норм и правил. В рамках этого подхода отказ системы в конструкций рассматривается как случайное событие, а надежность и устойчивость - как вероятность безотказной работы конструкций. Для систем с неизменными во времени внешними нагрузками и свойствами эффективным инструментом анализа выступают статические модели. Несмотря на совершенствование техники и технологии проблема оценки надежности и устойчивости конструкций при случайных нагрузках остается крайне актуальной из-за усложнения условий эксплуатации и роста ответственности объектов строительства. Научная новизна данного исследования заключается в следующем: впервые получены приближенные аналитические зависимости для определения надежности и долговечности упругих систем конструкций, одновременно учитывающие случайный характер как амплитуды переменных напряжений, так и предела выносливости строительных материалов; предложена новая инженерная методика вероятностного расчета и проектирования многоэлементных механических систем учитывающая эффект зависимых отказов элементов конструкций, что отличает её от классических схем с независимыми событиями; в отличие от ранее опубликованных автором базовых положений в настоящей работе обобщена и адаптирована теория усталостного разрушения для практических задач проектирования, а также впервые установлены явные взаимосвязи между статическими моделями надежности и многофакторными случайными параметрами конструкций. Разработанные в статье методы и формулы доведены до инженерных алгоритмов, что создает практическую основу для надежного проектирования сложных несущих систем конструкций.*

Ключевые слова: *надежность; упругие системы; напряженное состояние; предел прочности; долговечность.*

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1 INTRODUCTION

Reliability analysis of elastic systems-ranging from civil infrastructure components to complex mechanical assemblies-has fundamentally shifted from traditional deterministic methods (such as allowable stress design and semi-probabilistic limit state approaches) to advanced structural reliability theory. In modern engineering, reliability is defined as the time-dependent or static probability of failure-free operation under uncertain operational environments. While standard codebased probabilistic approaches account for safety margins via partial safety factors, they often fail to capture the complex, multi-source stochasticity inherent in real-world operating conditions. Despite recent advances in fatigue and structural reliability assessment, several critical limitations persist in contemporary probabilistic models: Over-simplified Loading Models: Most current analytical methods evaluate random fatigue loading assuming stationary or uncoupled stress amplitudes. However, under combined random loadings, determining the exact joint probabilistic characteristics of induced stresses remains an analytical challenge. Uncertainty in Material Parameters: Existing endurance limit models typically treat fatigue strength parameters as deterministic or uncoupled from loading variations. Relationships for evaluating reliability under arbitrary combinations of probability distributions (governing both acting stresses and material endurance limits) are still largely missing or restricted to narrow empirical bounds. Multi-Element Systems and Dependent Failures: Conventional system-level reliability models frequently rely on the assumption of statistically independent element failures. In complex elastic structures, failure events are inherently correlated due to shared stress fields and load redistributions a phenomenon that standard engineering algorithms fail to model efficiently. Lack of Practical Engineering Tools: While complex Monte Carlo simulations can solve high-dimensional stochastic problems, they are computationally prohibitive for routine design optimization. There remains a distinct lack of explicit analytical and approximate algorithms tailored for complex stress states and stability boundary analysis. Research Gap: Consequently, a clear scientific gap exists between general probabilistic safety concepts and computationally efficient, analytical reliability models. Existing approaches do not provide an integrated framework capable of handling simultaneous random variations in stress amplitudes, material endurance limits, and dependent multi-element failure modes without resorting to resource-intensive numerical simulations. Objective and Tasks: To address this gap, this study aims to develop and refine analytical methods for evaluating the reliability and long-term durability of elastic systems with random parameters under complex static and fatigue loading conditions. To achieve this objective, the following specific tasks are pursued: Derive explicit approximate analytical equations for assessing fatigue reliability and service life, accounting for joint random variations in variable stress amplitudes and endurance limits. Formulate a probabilistic calculation methodology for multielement elastic systems that explicitly incorporates dependent failure mechanisms. Establish a simplified, computationally robust engineering algorithm for the preliminary design and reliability verification of mechanical systems operating under complex stress states [1].

2 MATERIALS AND METHODS

Considering that for many machine parts and structural components the limiting state is taken as the state of failure due to fatigue, the fatigue curve is represented by the following relationship. The fatigue behavior of most construction materials can be adequately described by the following analytical relationship,

$$N \leq N_0 \text{ when } S_a^m N = R_{-1}^m N_0; \quad N > N_0 \text{ when } N = \infty, \quad (1)$$

where m – is a material constant;

R_{-1} - endurance limit of a laboratory specimen;

N_0 - baseline number of cycles;

S_a - amplitude of a fully reversed stress cycle;

N - number of cycles to failure.

For specific machine parts, instead of R_{-1} , the endurance limit of the part $R_g = R_{-1}/K_{-1}$ is used, calculated with consideration of the overall endurance limit modification factor. For a non-symmetric stress cycle, instead of S_a , the amplitude of the equivalent fully reversed stress cycle is used

$$S_{np} = S_a + (\psi_\sigma/K_{-1})S_m. \quad (2)$$

where R_{-1} - endurance limit of a laboratory specimen;

S_a - amplitude of a fully reversed stress cycle;

ψ_σ - the material sensitivity coefficient.

Here, the material sensitivity coefficient to cycle asymmetry is determined by the following formula

$$\psi_\sigma = (2R_{-1} - R_0)/R_0. \quad (3)$$

where R_{-1} - endurance limit of a laboratory specimen;

S_a - amplitude of a fully reversed stress cycle;

ψ_σ - the material sensitivity coefficient.

For carbon steels $\psi_\sigma = 0,1 \div 0,2$; and for alloy steels $\psi_\sigma = 0,05 \div 0,35$.

In the case of a complex stress state, instead of S_a , the amplitude of the equivalent stress is used, determined according to one of the failure criteria [3, 7]. Thus, in the general case, the fatigue curve can be written as follows:

$$N \leq N_0 \text{ when } N = N_0(R_g/S_a)^m; \quad N > N_0 \text{ when } N = \infty, \quad (4)$$

when experimentally determining the fatigue curve, a large scatter of experimental data is observed. Due to this significant scatter, a large number of specimens must be tested, and a complete probabilistic diagram is constructed. The analytical description of this diagram may vary, but the following expression has the strongest theoretical justification:

$$F = 1 - \exp \left[- \left(\frac{S - R_{-1}}{S_c} \right)^\alpha \left(\frac{N - N_0}{N_c} \right)^\beta \right], \quad (5)$$

where α , β , N_c , S_c are distribution parameters determined from experimental data.

The probabilistic diagram can be represented as a family of fatigue curves in $S - N$ coordinates corresponding to different failure probabilities P ; or as lifetime distribution curves in $P - N$ coordinates corresponding to different stress levels S ; finally, it may also be represented as curves $P - \sigma$ coordinates corresponding to different service lives N .

The dependence $S - N$ is usually plotted for $P = 0,5$ and represents the median value of the endurance limit. Sometimes it is constructed for a specified reliability level $P = H_A$ and used in the design of components with a prescribed reliability.

Studies [4, 2] have shown that the distribution of the number of cycles to failure is well described by a logarithmically normal law. The most important relationship for reliability calculations is the latter dependence $P - R$, that is, the distribution law of the endurance limit corresponding to different test bases. Numerous experimental data indicate that this dependence can be assumed to follow either a normal or a lognormal distribution. To obtain experimental data on the distribution of the endurance limit, tests are carried out at a baseline number of cycles. By processing the test results,

the empirical distribution function $F(R_{-1})$ is obtained (probity method), or the numerical distribution characteristics $\bar{R}_{-1}$, σ_R are determined (staircase method).

When calculating the reliability of structural components, it is necessary to know the probabilistic characteristics of fatigue resistance, taking into account the factors that affect the endurance limit of real elements. Conducting tests on a large number of real structural elements seems impractical. Therefore, computational methods for extrapolating the results of tests on samples to real structural elements are of utmost importance. Such capabilities are provided by the statistical theory of similarity in fatigue failure. As the principal similarity criterion, the ratio of the perimeter L of the critical $L/\bar{G}$, or its most highly stressed part, to the relative gradient G of the first principal stress in this section is considered. The quantity $L/\bar{G}$ is proportional to the stressed volume in which crack initiation is possible. The definition of this quantity can be found in the specialized literature [4].

According to this theory, the distribution of the endurance limit of a structural elements $f_R(R_g)$ is approximated by a normal law under the condition that it coincides with the initial lognormal distribution of the random variable $S_{\max}, U$, obtained from the original Weibull law, at two points - at the probabilities 0.5 and 0.997. Here, $S_{\max}$ is the maximum stress in the component, and U is the minimum value of the endurance limit. In this case, the median value R_g and the coefficient of variation k_{R_g} of the endurance limit of the structural elements are determined by the following formulas [4].

$$\bar{R}_g = \varepsilon_{\infty} \bar{R}_{-1} (1 + l); \quad k_{R_g} = 0,5(1 - 10^{-2\sigma}) l / (1 + l), \quad (6)$$

$$\text{where } l = \frac{1 - \varepsilon_{\infty}}{\varepsilon_{\infty}} \left(\frac{1}{88,3} \cdot \frac{L}{G} \right)^{-\nu},$$

ν - is an empirical coefficient,

ε_{∞} - scale factor for large-sized components,

σ - the standard deviation of the logarithm of the quantity $S - U$.

Formula (6) provides the possibility of accurately describing the regularities of fatigue failure in components with different geometric shapes and loading conditions by selecting these three parameters and the mean endurance limit value of the specimen $\bar{R}_{-1}$. In approximate calculations, the values $\varepsilon_{\infty} = 0,5$, $\nu = 0,1$, $\sigma = 0,045 \div 0,05$ are taken as; for structural steels, and $\sigma = 0,05 \div 0,06$ for alloys.

To calculate the reliability of a structural elements with respect to fatigue failure, it is necessary to know the distribution law of the amplitudes of the design stresses $f(S)$. At the design stage, for simple loading cycles, this distribution can be determined by transforming probability density functions or by solving problems of statistical dynamics. Thus, if the stress represents a harmonic process with a random amplitude, or a narrow-band stationary Gaussian process, the probability density function of the amplitude follows the Rayleigh distribution.

$$f(S) = (S/\sigma_S^2) \exp(-S^2/2\sigma_S^2), \quad (7)$$

where σ_S^2 - stress variance, S - design stress.

At the design stage, the probabilistic characteristics of the load are usually specified. In this case, by solving the problem of statistical dynamics, the correlation function or the spectral density of displacements is determined, and subsequently those of the design stresses. Then the distribution law of stress amplitudes can be obtained using the method of maxima, applying Rice's formula for the maxima of a Gaussian random process [6, 5, 11].

3 RESULTS AND DISCUSSION

The loading characteristics can be obtained by simulation modeling or by full-scale strain-gauge measurements during structural elements. From the oscillogram or the stress sample, an array of random amplitudes S_i is generated, and a histogram is constructed. Next, the most appropriate theoretical distribution law for the amplitude is selected.

If the stress is random not only in amplitude but also in the asymmetry coefficient, the process must first be schematized. The purpose of this schematization is to obtain the distribution function of stress amplitudes that is equivalent to the given random process in terms of the amount of fatigue damage produced. Methods of schematization include the methods of maxima, ranges, and full cycles. It is generally accepted that the method of full cycles provides the most acceptable results. In this method, a framework of a correlation table linking the extrema of each stress cycle is constructed based on oscillograms. The actual process is then replaced by an equivalent symmetric process with random amplitudes, and a histogram is constructed from this sample. A theoretical distribution law for the amplitude is then selected based on this histogram. A detailed scheme of the implementation of this method and the corresponding calculation formulas can be found in [5, 8, 10].

If the stress is a stationary Gaussian process, the following expressions can be used to determine the probability density function of the amplitudes [6, 5].

$$f(S) = \begin{cases} k_0^2 S \exp(-k_0^2 \alpha^2 S^2 / 2) & S < S_* \\ k_0^{-1} C S \exp(-S^2 / 2) & S > S_* \end{cases} \quad (8)$$

where k_0 - the initial ratio of the mean number of extrema to the mean number of zero crossings,
 S - design stress.

$$\alpha = \sqrt{2(a+3b)/ab}; \quad C = k_0^{-a/b} \exp(S_*^2/2); \quad S_* = \sqrt{a \ln k_0} / k_0. \quad (9)$$

To determine a and b , the following two equations are used:

$$k_0^{-a/b} = 1 - \frac{ab}{2k_0(a+b)} \left[1 - k_0^{-(a+b)/b} \right] = k_0 \left\{ 1 - \frac{ab}{2(a+3b)} \left[1 - k_0^{(a+3b)/b} \right] \right\}. \quad (10)$$

where k_0 - the initial ratio of the mean number of extrema to the mean number of zero crossings,
 S - design stress.

The graphs and tables of the dependence of the parameters on k_0 are presented in the work [6].

Let us consider the calculation of reliability with respect to fatigue strength under regular random loading. In this case, the reliability is equal to the probability that the random amplitude of the design stresses does not exceed the random value of the endurance limit, or its value corresponding to a limited number of cycles. The general solution to such problems, as well as the solutions for various combinations of applied and limiting stresses, was given in the previous section. Thus, for the normal distribution law of limiting and applied stresses, the reliability is equal to

$$H = \Phi(t), \quad (11)$$

here $t = (n_N - 1) / \sqrt{n_N^2 k_{Rg}^2 + k_S^2}$, $n_N = \bar{R}_{gN} / m_S$, $k_{Rg} = \sigma_{Rg} / \bar{R}_g$, H - system reliability.

Using expression (4), we obtain

$$N > N_0 \text{ when } n_N = \bar{R}_g / m_S = n_0; \quad N \leq N_0 \text{ when } n_N = n_0 (N_0 / N)^m. \quad (12)$$

where N_0 - baseline number of cycles,

N - number of cycles to failure.

These formulas make it possible to calculate the distribution function of the component's service life, that is, the dependence of reliability on the lifetime N .

The determination of reliability with respect to fatigue failure will be carried out using the corrected linear damage accumulation hypothesis, according to which the condition for fatigue failure after N cycles has the following form

$$a = N/N_p(S), \quad (13)$$

where $N_p(S)$ - S the number of cycles to failure at the given stress level.

For random variations of stress, taking into account expression (4), we have

$$a = N \int \frac{f(S)}{N_p(S)} dS = \frac{N}{N_0 R_g^m} \int_{R_g}^{S_m} S^m f(S) dS; \quad (14)$$

$$a = \left[\int_{R_g/2}^{\infty} S f(S) dS \int_{R_g/2}^{\infty} f(S) dS - \bar{R}_g^2 \right] / (S_m - \bar{R}_g^2). \quad (15)$$

Under block loading of components, $f(S)$ is determined for the block, while $N = \lambda v_\sigma$ is the number of loading cycles in the block, and λ is the number of blocks. Here, S_m is the maximum amplitude value, which can be obtained by specifying the probability that this value will not be exceeded, H_{sm} . Then S_m is determined as the root of the equation

$$H_{sm} = \int_{-\infty}^{S_m} f(S) dS = F(S_m), \quad (16)$$

That is, S_m is equal to the quantile of the distribution corresponding to the probability H_{sm} .

To calculate the coefficient a using expression (15), we introduce the following notations:

$$U = \int_{R_g/2}^{\infty} S f(S) dS \sigma_S \int_{R_g/2}^{\infty} f(s) dS, \quad x_0 = R_g/\sigma_S, \quad b = S_m/\sigma_S \quad (17)$$

and we write it as $a = (U - x_0/2)/(b - x_0/2)$. If the result of the calculation gives $a < 0,2$, then $a = 0,2$ should be set equal to.

Now, from equation (14), the service life (resource) of the component can be determined

$$N = a N_0 R_g^m / I \quad (18)$$

where

$$I = \int_{R_g}^{S_m} S^m f(S) dS. \quad (19)$$

The integrals (17) and (19) can be evaluated using the tabulated $P(x, n)$ distribution (Pearson distribution) χ^2 . In the work [4], expressions are given for the integral (19) with an infinite upper limit for various distributions $f(S)$. Using these results, the following expressions for integrals (17) and (19) can be obtained. Rayleigh distribution

$$I/R_g^m = 2^{m/2} \Gamma(1 + m/2) [Q(x_0^2, m + 2) - Q(b^2, m + 2)]/x_0^m; \quad (20)$$

$$U = \sqrt{\pi/2} Q(x_0^2/4, 3)/Q(x_0^2/4, 2), \quad (21)$$

here $Q(x, n) = 1 - P(x, n)$ - failure probability,
 P –probability of failure-free operation.

The right tail of the normal distribution:

$$I/R_g^m = 2^{m/2} \Gamma[(m+1)/2] [Q(x_0^2, m+2) - Q(b^2, m+2)] / \sqrt{\pi x_0^m}; \quad (22)$$

$$U = \sqrt{\pi/2} Q(x_0^2/4, 2) / Q(x_0^2/4, 1); \quad b = S_m/\sigma_S; \quad x_0 = R_g/\sigma_S. \quad (23)$$

where Q –failure probability,
 P –probability of failure-free operation.
 Exponential distribution

$$I/R_g^m = \Gamma(m+1) [Q(2x_0, 2m+2) - Q(2b, 2m+2)] / x_0^m; \quad (24)$$

$$U = Q(x_0, 4) / Q(x_0, 2); \quad b = S_m/\sigma_S; \quad x_0 = R_g/\sigma_S. \quad (25)$$

In these formulas, $\Gamma(\pi)$ - is the complete gamma function, Q –failure probability.

If the endurance limit is a random variable, then in all formulas it should be replaced by its median value $R_g - \bar{R}_g$. Then, using expression (18), the mean service life of the component can be determined.

To calculate reliability, from equation (18) we determine the threshold value of the endurance limit R_* , which corresponds to a service life of N cycles and the specified loading. If the actual endurance limit R_g exceeds R_* , then the structural elements will not fail within N cycles. Consequently, the reliability of the structural element with respect to fatigue failure is equal to the probability of this event and is determined by the following expression

$$H = P(R_g > R_*) = \int_{R_*}^{\infty} f_R(R_g) dR_g = 1 - \Phi(\gamma), \quad (26)$$

here $\gamma = (R_* - \bar{R}_g) / \sigma_g$; Φ - the tabulated distribution function of the normal law.

The transcendental equation (18) is solved either graphically or using a computer. For practical calculations, it is useful to construct in advance a plot of the dependence of x_0 on N/N_0 for various materials and loading conditions

$$N/N_0 = aR_g^m / I = f(x_0, m, b). \quad (27)$$

Graphs of such dependencies for the truncated normal distribution of stress amplitudes at $m = 9$ and various values of b are shown in (Figure 1). Similar graphs for the Rayleigh distribution of stress amplitudes at $b = 4$ and different values of m are shown in (Figure 2).

To determine the threshold value R_* for the material of the structural element under the given loading (m, N_0) and the specified service life ($b, f(S)$), the value N is obtained from the graph, and $R_* = x_0 \sigma_S$ is subsequently determined from it. Then, using formula (26), the fatigue reliability of the structural element is calculated. The results obtained in this section were published by the author in the article [9].

Approximate methods for calculating the reliability of elastic systems. The proposed method is based on replacing an arbitrary complex distribution law of the design stress with a weighted sum of normal distributions in the form of expression

$$f_S = \sum_{i=1}^n P_i f_{Hi}(S) = \sum P_i f_H(m_i, \sigma_i) \quad (28)$$

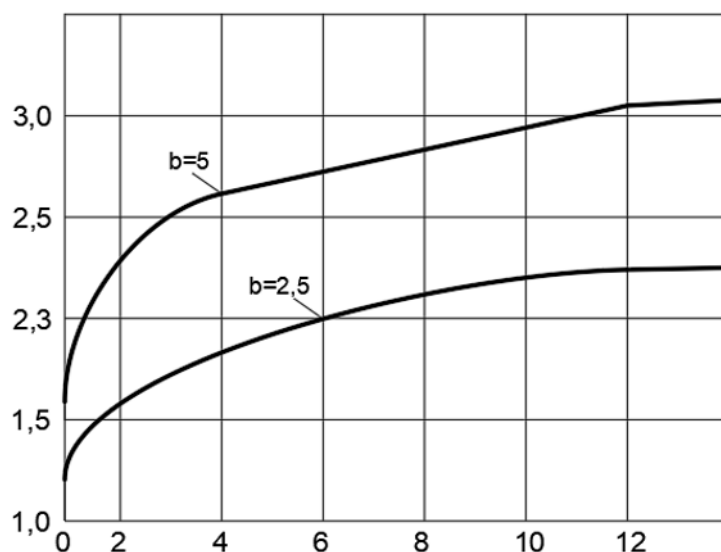


Figure 1 - Graphs for determining the threshold value of the endurance limit under the truncated normal amplitude distribution [Authors' material].

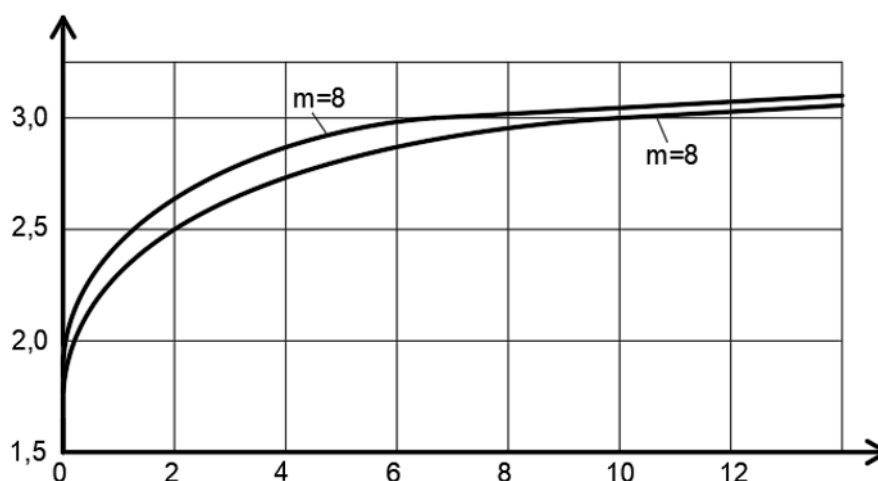


Figure 2 – Graphs for determining the threshold value of the endurance limit under the Rayleigh amplitude distribution $b = 4$ [Authors' material].

The specific form of the formulas for calculating the reliability of elastic systems depends on the distribution law of the limiting stress. For a normal distribution law, applying formulas $n = m_R/m_S$ and (28), we obtain

$$H = \sum P_i \Phi(t_i), \quad t_i = \frac{m_R - m_i}{\sqrt{\sigma_R^2 + \sigma_i^2}} \quad (29)$$

If the limiting stress is distributed according to the Weibull law

$$\begin{aligned} f(x) &= (b/a)(x - x_0)^{b-1} \exp[-(x - x_0)^b/a] & \text{при } x \geq x_0; \\ f(x) &= 0 & \text{при } x < x_0. \end{aligned} \quad (30)$$

Then one can make use of the results from.

Then $P = \Phi(t_0) + J$. Taking the above relationship into account, we obtain the following expression for calculating the reliability

$$H = \sum_{i=1}^n P_i [\Phi(t_i) + J_i], \quad (31)$$

where the integrals J_i are determined by the expression $J = \frac{A}{\sqrt{2\pi}} \int_0^\infty \exp[-y^\beta - (Ay + t_0)^2/2] dy$.

The expression for determining the probability of failure-free operation when the limiting stress follows a gamma distribution is obtained from formula $P = (C/\sqrt{2\pi}) \int_{-t_0}^\infty e^{-t^2/2} dt = C\Phi(t_0)$, and takes the form

$$H = \sum_{i=1}^n P_i C_i \Phi(t_i), \quad (32)$$

here $a = \alpha + 0,5$, $\delta_i = \sigma_i/\beta$, $t_i = -(\delta_i + m_i/\sigma_i - 2\sqrt{a})/\sqrt{2}$, $C_i = \frac{\sqrt{\pi} \delta_i^{a+1} \exp(a - 2\delta_i\sqrt{a} + \delta_i^2/2)}{2^{a/2-1/4} \Gamma(a/2+3/4)}$.

If the distribution law of the limiting stress is given in the form of a histogram or has a complex form, it can also be represented as a composition of normal distributions

$$f_R = \sum_{j=1}^m P_{Rj} f_H(m_{Rj}, \sigma_{Rj}). \quad (33)$$

In this case, using formula $n = m_R/m_S$, we obtain

$$H = \sum_{i=1}^n \sum_{j=1}^m P_{Si} P_{Rj} \Phi(t_{ij}), \quad t_{ij} = \frac{m_{Rj} - m_{Si}}{\sqrt{\sigma_{Rj}^2 + \sigma_{Si}^2}}. \quad (34)$$

The approximate methods also include the graphical method for determining the probability of failure-free operation. This method is used when there is no justification for assuming any specific distribution law for stress or strength, but a sufficiently large volume of experimental data is available. Let us introduce the following notations

$$G = \int_S^\infty f_R(R) dR = 1 - F_R(S), \quad H = \int_{-\infty}^t f_S(S) dS = F_S(t). \quad (35)$$

Then $dH = f_S(S) dS$. It is obvious that the values of H lie within the interval from 0 to 1.

Substituting these expressions into formula $H = P = \int_{-\infty}^R f_S(S) dS = F_S(R)$, we obtain $P = \int_0^1 G dH$.

This expression shows that the reliability is equal to the area under the curve of the dependence of H on G . Using the available data on strength and stress, estimates of $\bar{F}_R(R)$ and $\bar{F}_S(S)$ can be determined, and tables or empirical curves of these estimates can be constructed. Then, for various values of S_i , the desired functions can be easily obtained

$$G_i = 1 - \bar{F}_R(S_i), \quad H_i = \bar{F}_S(S_i). \quad (36)$$

Using these points, the dependence of H on G can be easily constructed. The area under this curve, taken from the graph, gives the desired probability. The number of publications on this relevant topic in scientific periodicals and conference proceedings continues to grow [17, 18, 19, 20, 21].

Example. Ten measurements of stress were performed under simulated operating conditions. Based on these data, the stress distribution function $\bar{F}_S$ was estimated. During the strength analysis of the element, 14 strength values were obtained, on the basis of which the strength distribution function was estimated. All the data are presented in (Table 1).

Table 1 - Distribution functions of strength [Authors' material].

№	Stress S , MPa	$\bar{F}_S$	Strength R , MPa	$\bar{F}_R$
1	207.5	0,1	338	0,07
2	236	0,2	343	0,14
3	245	0,3	354	0,21
4	265	0,4	359	0,28
5	265,5	0,5	360	0,35
6	275	0,6	362	0,43
7	292	0,7	368	0,5
8	300	0,8	370	0,57
9	337,5	0,9	371	0,64
10	375	1,0	373	0,71
11	-	-	382	0,78
12	-	-	385	0,85
13	-	-	400	0,93
14	-	-	420	1,00

Using these data, we construct the empirical curves $\bar{F}_S(S)$ and $\bar{F}_R(R)$. By specifying various values of S_i , we obtain from the graphs the corresponding values of $\bar{F}_S(S_i)$ and $\bar{F}_R(S_i)$. We then determine G_i , H_i and construct the graph of the dependence H G . A direct measurement of the area under this curve gives the reliability, which was found to be equal to 0.988.

When analyzing the following sources [12, 13, 14, 15, 16] during the reconstruction and modernization of building structures of various facilities under probabilistic loads with respect to fatigue failure, the reliability of the system must be preserved.

4 CONCLUSIONS

1. Based on the statistical laws of fatigue failure, formulas for determining the reliability of fatigue resistance have been obtained.
2. On the basis of the statistical characteristics of fatigue cracking and failure, improved methods have been developed for calculating the durability and reliability of structural components and machine parts subjected to random cyclic loading.
3. For various combinations of the distribution laws of design and limiting stresses, relationships have been derived that account for the connection between the geometric characteristics of the structure and its reliability index.
4. Based on the development of probabilistic methods for evaluating the reliability of elastic systems, problems of determining the load carrying capacity of compressed flexible rods, taking into account groups of random factors, have been solved.
5. This paper improves the results presented in studies [1, 2, 6, 9, 10, 13].

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The authors declare no conflict of interest.

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B.B. Aubakirova – Literature review, data collection and systematization, visualization (graphics).

M.A. Tanirbergenova – Mathematical modeling, calculations.

Sh.B. Toleubayeva – Data processing / Result processing.

U.T. Begaliev – Literature review, data collection and systematization.

B.O. Uskambaeva – Writing – review and editing.

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