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RESEARCH ARTICLE

DEVELOPMENT OF A MATHEMATICAL MODEL OF HEAT TRANSFER THROUGH A MULTILAYER ENCLOSING STRUCTURE WITH AIR LAYERS

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Abstract. *A mathematical model of heat transfer in an innovative external building enclosure, comprising both ventilated and non-ventilated air layers, has been developed in this study. The structural complexity of the considered system necessitates the simultaneous consideration of all three modes of heat transfer: thermal conductivity (conductive transfer), convection, and thermal radiation. The model incorporates boundary conditions on both the interior and exterior surfaces of the enclosure, allowing for the influence of climatic factors and building operation conditions to be taken into account. For the conductive heat transfer, the thermal characteristics of the enclosure layers are assigned based on the physical and mechanical properties of the construction materials. For the layer consisting of a fully enclosed air gap, the model uses the equivalent thermal conductivity value that accounts for heat transfer in a stationary gas medium. In the case of the ventilated air layer, the energy equation is applied to describe the heat exchange processes involving moving air, including the interaction between the airflow and bounding surfaces. As a result, a mathematical model in the form of a system of heat transfer equations adapted to the multilayer structure of the enclosure has been constructed, accompanied by the appropriate boundary conditions. This enables accurate simulation of temperature fields and heat flows depending on the parameters of the construction and the external environment. The developed model can be used in engineering calculations for design tasks, thermal optimization, energy audits, and evaluation of the efficiency of various enclosure systems. Furthermore, the model can be integrated into building information modeling (BIM) software and used in energy efficiency certification systems. The presented results contribute to improving approaches to analyzing the thermal behavior of building structures and to the development of sustainable construction principles.*

Keywords: *mathematical model, heat transfer, multilayer enclosure, air layer, boundary conditions.*

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ҒЫЛЫМИ МАҚАЛА

АУА ҚАБАТТАРЫ БАР КӨП ҚАБАТТЫ ҚОРШАУ КОНСТРУКЦИЯСЫ АРҚЫЛЫ ЖЫЛУ ӨТКІЗУДІҢ МАТЕМАТИКАЛЫҚ МОДЕЛІН ЖАСАУ

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Аңдатпа. *Бұл жұмыста бір мезгілде желдетілетін және желдетілмейтін ауа қабаттары бар сыртқы инновациялық қоршау конструкциясындағы жылу алмасудың математикалық моделі әзірленді. Қарастырылып отырған жүйенің конструктивтік күрделілігі жылу алмасудың үш механизмі – жылуөткізгіштік (кондуктивті тасымалдау), конвекция және сәулелік жылу алмасуды – бір уақытта ескеруді талап етеді. Модельде қоршау конструкциясының ішкі және сыртқы беттерінде шекаралық шарттар берілген, бұл климаттық факторлар мен ғимараттың пайдалану жағдайларының әсерін есепке алуға мүмкіндік береді. Кондуктивті жылуөткізгіштікті сипаттау кезінде қоршау қабаттарының жылутехникалық сипаттамалары құрылыс материалдарының физика-механикалық қасиеттеріне сәйкес қабылданады. Толық жабық ауа қабатынан тұратын қабат үшін модельде қозғалмайтын газ ортасындағы жылу беруді ескеретін эквивалентті жылуөткізгіштік мәні қолданылады. Ал желдетілетін ауа қабаты үшін жылу алмасу процестерін, оның ішінде ауа ағыны мен шекаралық беттердің арасындағы өзара әрекеттесуді сипаттау үшін энергия теңдеуі қолданылады. Нәтижесінде көпқабатты қоршау құрылымына бейімделген және сәйкес шекаралық шарттармен толықтырылған жылу тасымалдау теңдеулерінің жиынтығы түріндегі математикалық модель әзірленді. Бұл температуралық өрістер мен жылу ағындарын конструкция параметрлері мен сыртқы орта жағдайларына байланысты дәл модельдеуге мүмкіндік береді. Дайындалған модель жобалау, жылутехникалық оңтайландыру, энергия аудиті және әртүрлі қоршау шешімдерінің тиімділігін бағалау есептерінде инженерлік есептеулерде қолданылуы мүмкін. Сонымен қатар, модельді ғимараттарды ақпараттық модельдеу (BIM) бағдарламалық кешендеріне біріктіруге және энергия үнемділік сертификаттау жүйелерінде қолдануға болады. Ұсынылған нәтижелер құрылыс конструкцияларының жылулық мінез-құлқын талдау тәсілдерін жетілдіруге және орнықты құрылыс қағидаттарын дамытуға ықпал етеді.*

Түйін сөздер: *математикалық модель, жылу беру, көп қабатты қоршау, ауа қабаты, шарттық жағдайлар.*

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НАУЧНАЯ СТАТЬЯ

РАЗРАБОТКА МАТЕМАТИЧЕСКОЙ МОДЕЛИ ТЕПЛОПЕРЕДАЧИ ЧЕРЕЗ МНОГОСЛОЙНУЮ ОГРАЖДАЮЩУЮ КОНСТРУКЦИЮ С ВОЗДУШНЫМИ ПРОСЛОЙКАМИ

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Аннотация. *В работе разработана математическая модель теплопередачи наружного инновационного ограждения, содержащего одновременно вентилируемую и невентилируемую воздушные прослойки. Конструктивная сложность рассматриваемой системы обуславливает необходимость одновременного учёта всех трёх механизмов теплопередачи: теплопроводности (кондуктивной теплопередачи), конвекции и лучистого теплообмена. В разработанной модели заданы граничные условия как на внутренней, так и на наружной поверхности ограждающей конструкции, что позволяет учитывать влияние климатических факторов и условий эксплуатации здания. При описании кондуктивной теплопередачи теплотехнические характеристики слоёв ограждения принимаются в соответствии с физико-механическими свойствами строительных материалов. Для слоя, представляющего собой полностью замкнутую воздушную прослойку, используется величина эквивалентной теплопроводности, учитывающая теплопередачу в неподвижной газовой среде. В случае вентилируемой прослойки в модели используется уравнение энергии, позволяющее описать процессы теплообмена с участием движущегося воздуха, включая взаимодействие между потоком и граничащими поверхностями. В результате построена математическая модель в виде набора уравнений теплопереноса, адаптированных к многослойной структуре ограждения и сопровождаемых соответствующими граничными условиями. Это обеспечивает возможность точного моделирования температурных полей и тепловых потоков в зависимости от параметров конструкции и внешней среды. Разработанная модель может быть использована при инженерных расчётах в задачах проектирования, теплотехнической оптимизации, энергоаудита и оценки эффективности различных ограждающих решений. Также возможна интеграция модели в программные комплексы для информационного моделирования зданий (BIM) и последующее применение в системах сертификации энергоэффективности. Представленные результаты способствуют совершенствованию подходов к анализу теплового поведения строительных конструкций и развитию принципов устойчивого строительства.*

Ключевые слова: *математическая модель, теплопередача, многослойное ограждение, воздушная прослойка, граничные условия.*

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CONFLICT OF INTEREST

The authors state that there is no conflict of interest.

АЛҒЫС/ҚАРЖЫЛАНДЫРУ КӨЗІ

Зерттеу Қазақстан Республикасы Ғылым және жоғары білім министрлігі Ғылым комитетінің гранттық қаржыландыруы аясында (жоба № AP22782896) «Қазақстанның климаттық жағдайын және динамикалық жүктемелерді ескере отырып ғимараттардың энергия үнемдейтін сыртқы қабырға конструкцияларын әзірлеу» тақырыбы бойынша жүргізілді.

МҮДДЕЛЕР ҚАҚТЫҒЫСЫ

Авторлар мүдделер қақтығысы жоқ деп мәлімдейді.

БЛАГОДАРНОСТИ/ИСТОЧНИК ФИНАНСИРОВАНИЯ

Исследование проведено в рамках грантового финансирования Комитета науки Министерства науки и высшего образования Республики Казахстан (проект № AP22782896) по теме: «Разработка энергоэффективных наружных стеновых конструкций зданий с учетом динамических воздействий в условиях регионов Казахстана».

КОНФЛИКТ ИНТЕРЕСОВ

Авторы заявляют, что конфликта интересов нет.

1 INTRODUCTION

The development of new energy-efficient multilayer building envelope structures is currently among the most prominent trends at the international level.

To address this issue, the scientific and engineering community is investigating various innovative structural solutions in the field of external enclosures. The relevance of this direction is driven by heat losses through building envelopes, where thermal energy consumption can reach up to 40%, simultaneously causing both economic and environmental concerns.

Given that building envelopes play a key role in establishing indoor climatic parameters, it is feasible to utilize an adaptive structural system, in which a two-channel air layer is proposed as an innovative solution. This configuration has a positive impact on thermal energy savings during both the winter and summer seasons. However, due to the technical complexity and the payback of economic investments – where cost increases reach nearly 50% – the practical implementation of such systems at the national level in the Republic of Kazakhstan presents certain challenges.

The use of double-skin glass façades equipped with integrated movable shading devices significantly enhances the energy performance of buildings, particularly in comparison to opaque walls. As an adaptive solution for moisture removal in multilayer external enclosures, the incorporation of ventilated air channels into the envelope structure is feasible, providing high thermal insulation properties for the panels and resulting in substantial energy savings.

Given the above circumstances, the development of new energy-efficient structures is currently being pursued actively, employing various research methods ranging from experimental studies to finite element modeling using modern software platforms.

Accordingly, the objective of this study is to develop a mathematical model of heat transfer through a new energy-efficient multilayer building envelope with air layers, which will subsequently enable the application of the developed model in theoretical calculations and engineering analysis.

2 LITERATURE REVIEW

The studies by **Xian et al. (2025)** and **Manzueta et al. (2024)** demonstrate that external building envelope structures account for a significant share of heat loss in buildings, highlighting the need for new technological solutions.

In the research conducted by **Astorqui & Porras-Amores (2017)**, as well as **Dimoudi et al. (2004)**, façade systems with double air chambers are examined, where the movement of air masses is controlled.

The positive effect of a ventilated air layer in combination with dynamic insulation layers is also noted by **Pasquay (2004)**, with evidence demonstrating reduced cooling costs for buildings during warm seasons.

An analysis of the efficiency of double-glazed glass façades with integrated movable shading elements is presented in the studies by **Baldinelli (2009)** and **Bessoudo et al. (2010)**. Such solutions are capable of enhancing the energy performance of buildings throughout their entire service life.

The studies by **Nizovtsev et al. (2020)** and **Gagliano & Aneli (2020)** emphasize the importance of ventilated channels in multilayer panels, which not only effectively remove moisture but also maintain stable thermal insulation characteristics under conditions of high humidity.

Mathematical modeling of heat transfer through multilayer structures with air gaps is examined in the work of **Zhangabay et al. (2023)**, where the efficiency of heat-reflecting screens was analyzed using the ANSYS software package.

3 MATERIALS AND METHODS

In this study, a model of an energy-efficient external wall structure featuring both ventilated and non-ventilated air layers has been developed. Traditional calculation methods for determining heat transfer, as presented in the regulatory document **(SP RK 2.04-107-2022)**, are found to be inapplicable due to the presence of an additional enclosed air layer, which serves as a novel structural solution **(SP RK 2.04-107-2022, 2022)**. Consequently, there is a need for the initial development of a mathematical model of such a structure for further research.

To achieve this goal, the geometric and thermal characteristics of the studied structure are presented in Figure 1 and Table 1. Boundary conditions related to indoor microclimate parameters and external climatic influences may subsequently be adopted in accordance with relevant standards **(SP RK 2.04-01-2017, 2017; GOST 30494-2011, 2011)**.

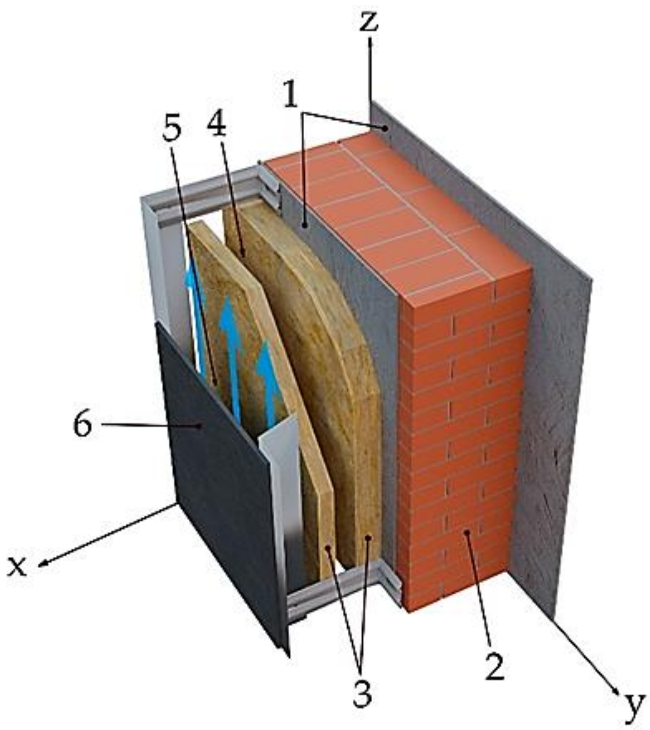

Figure 1 – Multilayer structure of the external building envelope

Table 1

Thermal characteristics of the layers of the multilayer external building envelope structure **(SP RK 2.04-107-2022, 2022)**.

Layer number	Description	Density, kg/m³	Thermal conductivity coefficient, W/(m·°C)	Vapor permeability, mg/(m·h·Pa)
1	Cement-sand plaster	1800	0,76	0,09
2	Masonry of ceramic bricks	1400	0,58	0,14
3	Extruded polystyrene foam or basalt wool insulation with a density of not less than 110 kg/m³	25	0,03-0,035	0,005
4	Enclosed air cavity	–	–	–
5	Ventilated air cavity	–	–	–
6	Porcelain stoneware	2800	3,49	0,008

4 RESULTS AND DISCUSSION

When constructing the mathematical model of heat transfer through the considered multilayer building envelope structure, the layers are described as homogeneous isotropic media with an equivalent thermal conductivity. This conductivity accounts for conductive, convective, and radiative components of heat transfer through the respective layer of the envelope structure. Such an approach allows the heat transfer problem to be considered in a one-dimensional steady-state formulation.

Let the spatial x-axis be directed from the interior toward the exterior, with the origin placed at the inner surface of the building envelope. Then, introducing the notation δ_i for the thickness of the *i*-th layer of the envelope structure, the following expressions can be written:

$$x_0 = 0,\ x_k = \sum_{i=1}^{k} \delta_i,\ x_n = \sum_{i=1}^{n} \delta_i, \tag{1}$$

For each layer, except for the ventilated air cavity, the one-dimensional linear steady-state heat conduction equation can be written.

$$\lambda_k \frac{\partial^2 T}{\partial x^2} = 0, \qquad x_{k-1} < x < x_k, \qquad k = 1, \ldots n, \tag{2}$$

At the interface between layers, due to the ideal thermal contact, the conditions describing the continuity of temperature and heat flux are satisfied:

$$T\big|_{x=x_k-0} = T\big|_{x=x_k+0}, \qquad k = 1, \ldots n-1, \tag{3}$$

$$\lambda_k \frac{\partial T}{\partial x}\bigg|_{x=x_k-0} = \lambda_{k+1} \frac{\partial T}{\partial x}\bigg|_{x=x_k+0}, \qquad k = 1, \ldots n-1. \tag{4}$$

Convective heat exchange between the building envelope and the surrounding air environment is described by the following boundary conditions:

on the inner surface of the building envelope

$$\lambda_1 \frac{\partial T}{\partial x}\bigg|_{x=0} = \alpha_{\text{int}}\left(T\big|_{x=0} - T_{\text{int}}\right), \tag{5}$$

on the outer surface of the building envelope

$$\lambda_n \frac{\partial T}{\partial x}\bigg|_{x=x_n} = \alpha_{ext}\left(T\big|_{x=x_n} - T_{ext}\right). \tag{6}$$

To simplify subsequent expressions, the following notations are introduced:

$T_o = T(0)$ – temperature at the inner surface of the building envelope;

$T_k = T(x_k)$ – temperature at the interface between layers numbered x_k and x_{k+1};

$T_n = T(x_n)$ – temperature at the outer surface of the building envelope.

The thermal conductivity coefficients for the layers without air cavities are assigned based on the initial data provided in Table 1.

For a layer consisting entirely of a closed air cavity, the equivalent thermal conductivity value must be used in equations (2) in place of λ_k. Let us denote this as $\lambda_{\text{э.в.}k}$.

Since the equivalent thermal conductivity implies a homogeneous isotropic medium, the heat flux density in the *x*-direction can be written as:

$$q = -\lambda_{\text{э.в.}k} \frac{\partial T}{\partial x}. \tag{7}$$

Due to the one-dimensional nature of the problem and the absence of internal heat sources, the partial derivative of temperature with respect to the spatial coordinate in expression (7) can be accurately represented as a finite difference, which allows expression (7) to be rewritten in the following form:

$$q = -\lambda_{\text{э.в.}k} \frac{T_k - T_{k-1}}{\delta_k} = \lambda_{\text{э.в.}k} \frac{T_{k-1} - T_k}{\delta_k}. \quad (8)$$

On the other hand, when considering the air cavity, the heat flux can be represented as the sum of two components – conductive-convective and radiative:

$$q = q_{\text{к}} + q_{\text{л}}. \quad (9)$$

According to **Lykov (1978)** and **Kutateladze (1990)**, in vertical air cavities between flat surfaces, conductive-convective heat transfer under natural convection conditions can be adequately described by mathematical models for stationary layers, using the concept of an effective thermal conductivity coefficient λ_{k}, which is defined as follows:

$$\begin{aligned} &\lambda_{\text{к}} = \lambda_{\text{в}} \quad \text{at} \quad \text{Ra} < 104, \\ &\lambda_{\text{к}} = \lambda_{\text{в}} \cdot 0{,}062 \cdot \text{Ra}^{1/3} \quad \text{at} \quad 10^4 < \text{Ra} < 10^4, \\ &\lambda_{\text{к}} = \lambda_{\text{в}} \cdot 0{,}22 \cdot \text{Ra}^{1/4} \quad \text{at} \quad 10^7 < \text{Ra} < 10^{10}, \end{aligned} \quad (10)$$

where $\text{Ra} = \dfrac{g\delta^3\beta\Delta T}{\nu}\text{Pr}$

The values of $\lambda_{\text{в}}$, ν and Pr expression (10) are taken at the averaged temperature and pressure within the air cavity.

Thus

$$q_{\text{к}} = -\lambda_{\text{к}} \frac{\partial T}{\partial x}. \quad (11)$$

The radiative component of the heat flux density, according to a number of studies, can be determined from the corresponding expression **(Zhangabay et al., 2023a; Zhangabay et al., 2023b; Zhangabay et al., 2024)**.

$$q_{\text{л}} = \varepsilon\sigma_0(T_{k-1}{}^4 - T^4{}_k), \quad (12)$$

Substituting (8), (11), and (12) into (9) and using the same finite difference representation of the temperature partial derivative for (13) as in (8), we obtain:

$$\lambda_{\text{э.в.}k} \frac{T_{k-1} - T_k}{\delta_k} = \lambda_{\text{к}} \frac{T_{k-1} - T_k}{\delta_k} + \varepsilon\sigma_0\left(T_{k-1}^4 - T_k^4\right), \quad (13)$$

from which we obtain

$$\lambda_{\text{э.в.}k} = \lambda_{\text{к}} + \frac{\delta_k \varepsilon\sigma_0\left(T_{k-1}^4 - T_k^4\right)}{T_{k-1} - T_k}. \quad (14)$$

To formulate the mathematical model of heat exchange in the ventilated air cavity, the energy equation is used, which in the general multidimensional unsteady case can be written as:

$$c\rho \frac{dT}{d\tau} = \lambda \Delta T + q_V, \quad (15)$$

In the ventilated air cavity, a clearly defined upward airflow is formed, with the absence of local vortices. Given the negligible viscous effects in the boundary layers near the walls enclosing the cavity, the air velocity can be assumed to be uniform across the cross-section of the cavity. Moreover, since the air temperature changes insignificantly along the flow direction within the cavity, the compressibility of air can be neglected in the energy equation, and its density can be considered constant.

Therefore, given that the air mass flow rate is constant, the air velocity can be assumed to be constant at every point within the ventilated cavity. In this case, by expressing the total time derivative in equation (15) through partial time derivatives, the one-dimensional steady-state form of the equation can be written as:

$$c\rho v\frac{\partial T}{\partial z} = \lambda\frac{\partial^2 T}{\partial z^2} + q_V\,, \tag{16}$$

The volumetric density of internal heat sources is defined based on the heat balance of the fluxes passing through the walls enclosing the cavity: the amount of heat entering an elementary volume of air in the cavity is equal to the difference between the amount of heat received from the indoor side and the amount of heat transferred through the outer cladding to the surrounding environment:

$$\delta Q_V = \delta Q_{int} - \delta Q_{ext}, \tag{17}$$

For an elementary volume of size $\delta_k\times\delta L\times\delta z$ (where δ_k is the thickness of the cavity, and δL and δz are the dimensions of the elementary volume in the horizontal and vertical directions, respectively), equation (17) can be written as:

$$q_V\cdot\delta_k\cdot\delta L\cdot\delta z = \alpha_п\cdot\delta L\cdot\delta z\,(T_{k-1} - T(z)) - \alpha_п\cdot\delta L\cdot\delta z\,(T(z) - T_k), \tag{18}$$

From relation (18), we obtain the expression for the specific volumetric density of internal heat sources.

$$q_V = \frac{\alpha_п}{\delta_k}\left(T_{k-1} + T_k - 2T(z)\right), \tag{19}$$

Since in the considered mathematical model the air velocity in the ventilated air cavity remains constant with height, and the temperature of the cavity's bounding walls is assumed to be constant, it follows from (18) that the air in the cavity heats up uniformly at every point. Consequently, the temperature distribution $T(z)$ is a linear function of the vertical coordinate. Therefore, the second derivative of air temperature with respect to the vertical coordinate is equal to zero, and equation (16), taking into account (19), can be rewritten as:

$$c\rho v\frac{dT}{dz} = \frac{\alpha_п}{\delta_k}\left(T_{k-1} + T_k - 2T(z)\right), \tag{20}$$

The initial condition for equation (20) is defined based on the fact that at the lowest point of the air cavity, air inflow occurs, and its temperature is equal to the ambient air temperature:

$$T(0) = T_{ext}, \tag{21}$$

The value of the heat transfer coefficient α_p can be obtained from well-known empirical correlations for a flat channel **(Lykov, 1978; Kutateladze, 1990; Isachenko, Osipova, and Sukomel, 1975)**:

$$\mathrm{Nu} = \mathrm{Nu}(\mathrm{Re}, \mathrm{Pr}), \tag{22}$$

where $\mathrm{Nu} = \alpha_п/(\delta_k\cdot\lambda)$.

The solution of the ordinary differential equation (20) with the initial condition (21) has the following form:

$$T(z) = \left[T_{\text{ext}} - \frac{T_{k+1} + T_k}{2} \right] \cdot \exp\left(-\frac{2\alpha_{\text{п}}}{\delta_{\text{k}} c \rho v} z \right) + \frac{T_{k+1} + T_k}{2}. \tag{23}$$

The average air temperature in the ventilated cavity can be determined as the mean integral value over the height:

$$T_{\text{п}} = \frac{1}{H} \int_0^H T(z) dz = \frac{1}{H} \left[1 - \frac{T_{\text{ext}} - \dfrac{T_{k+1} + T_k}{2}}{2\alpha_{\text{п}}} \delta_{\text{k}} c \rho v \cdot \exp\left(-\frac{2\alpha_{\text{п}}}{\delta_{\text{k}} c \rho v} H \right) \right], \tag{24}$$

The air temperature at the outlet of the ventilated cavity is obtained from expression (23) at $z = H$:

$$T_{\text{out}} = \left[T_{\text{ext}} - \frac{T_{k+1} + T_k}{2} \right] \cdot \exp\left(-\frac{2\alpha_{\text{п}}}{\delta_{\text{k}} c \rho v} H \right) + \frac{T_{k+1} + T_k}{2}, \tag{25}$$

The air velocity in the ventilated cavity can be determined based on the balance between the total pressure losses in the ventilated air cavity and the driving pressure (natural draught), which is caused by Archimedean forces arising due to the difference in air density within the ventilated cavity and at its outlet:

$$\Delta p^{\text{o}} = p_{\text{c}}, \tag{26}$$

Neglecting the pressure losses at the inlet and outlet, equation (26) for a flat vertical slot channel can be rewritten as:

$$\xi(H/2\delta_k)\rho_{\text{п}} v^2/2 = (\rho_{\text{ext}} - \rho_{\text{out}}) g H, \tag{27}$$

The air density at a given temperature in equation (27) can be obtained from the Clapeyron-Mendeleev equation:

$$\rho_i = p_i/(RT_i), \tag{28}$$

In which the thermodynamic pressure p_i can be taken as 1 atm. The heat flux balance within the cavity is described by the law of conservation of energy: the amount of heat entering the ventilated air cavity from the interior side Q_{int} is equal to the sum of the heat carried away by the ventilation air Q_{out}, the heat transferred by convection from the air cavity to the outer cladding Q_{ext},, and the heat transferred by radiation from the inner wall of the cavity to the outer wall $Q_{\text{л}}$:

$$Q_{\text{int}} = Q_{\text{out}} + Q_{\text{ext}} + Q_{\text{л}}. \tag{29}$$

For a unit length δL of the building envelope in the horizontal direction, these quantities can be expressed as:

$$Q_{\text{int}} = \alpha_{\text{п}}(T_{k-1} - T_{\text{п}}) \cdot H \cdot \delta L;\ Q_{\text{ext}} = \alpha_{\text{п}}(T_{\text{п}} - T_k) \cdot H \cdot \delta L; \tag{30}$$
$$Q_{\text{out}} = v\, \rho \cdot \delta_k \cdot \delta L \cdot c \cdot (T_{\text{out}} - T_{\text{ext}});\ Q_{\text{л}} = \varepsilon \cdot \sigma_0 \cdot (T_{k-1}^4 - T_k^4) \cdot H \cdot \delta L.$$

Substituting (30) into (29), we obtain:

$$\alpha_{\text{п}}(T_{k-1} - T_{\text{п}}) \cdot H = \alpha_{\text{п}}(T_{\text{п}} - T_k) \cdot H + v\, \rho \cdot \delta_k \cdot c \cdot (T_{\text{out}} - T_{\text{ext}}) + \varepsilon \cdot \sigma_0 \cdot (T_{k-1}^4 - T_k^4) \cdot H. \tag{31}$$

Thus, the mathematical model of the heat transfer phenomenon through the building envelope is described by the following set of equations:

1. For all layers except the ventilated air cavity, the heat flux is determined by equation (8). In this case, the thermal conductivity coefficient in equation (8) is either the corresponding thermophysical property of the material (for a solid layer) or the equivalent thermal conductivity coefficient calculated using formula (14).
2. Equations (24), (25), (27), (28), and (31) for the ventilated air cavity.
3. Boundary conditions (5), (6).

This mathematical model allows determining the temperature distribution T_k at the interfaces between layers, the air temperature within the ventilated cavity $T_п$, the outlet air temperature T_{ou}, and the air velocity in the ventilated cavity v. The temperature within each layer is found through linear approximation based on the temperatures T_k.

5 CONCLUSIONS

1. A mathematical model of heat transfer in an innovative external envelope structure featuring both ventilated and non-ventilated air layers has been developed.
2. The proposed model, due to its structural characteristics, takes into account all three modes of heat transfer: conductive, convective, and radiative.
3. Boundary conditions were defined for both the interior and exterior surfaces of the building envelope.
4. In modeling conductive heat transfer, the thermal properties were assigned according to the characteristics of the construction materials. For the layer consisting entirely of a closed air gap, the model utilizes an equivalent thermal conductivity value, while for the ventilated component of the envelope, the heat exchange model employs the energy equation.
5. As a result, a set of equations was developed that incorporates the boundary conditions, enabling accurate description of the external envelope structure and allowing the model to be applied in future calculation methodologies.

REFERENCES

1. Xian, Y., Wang, H., Zhang, Z., Yang, Y., & Zhong, Y. (2025). Driving factors and reduction paths dynamic simulation optimization of carbon dioxide emissions in China's construction industry under the perspective of dual carbon targets. *Environmental Impact Assessment Review, 112*, 107789. https://doi.org/10.1016/j.eiar.2024.107789
2. Manzueta, R., Kumar, P., Ariño, A. H., & Martín-Gómez, C. (2024). Strategies to reduce air pollution emissions from urban residential buildings. *Science of The Total Environment, 951*, 175809. https://doi.org/10.1016/j.scitotenv.2024.175809
3. Astorqui, J., & Porras-Amores, C. (2017). Ventilated façade with double chamber and flow control device. *Energy and Buildings, 149*, 471–482. https://doi.org/10.1016/j.enbuild.2017.04.063
4. Dimoudi, A., Androutsopoulos, A., & Lykoudis, S. (2004). Experimental work on a linked, dynamic and ventilated, wall component. *Energy and Buildings, 36*, 443–453. https://doi.org/10.1016/j.enbuild.2004.01.048
5. Pasquay, T. (2004). Natural ventilation in high-rise buildings with double facades: Saving or waste of energy? *Energy and Buildings, 36*, 381–389. https://doi.org/10.1016/j.enbuild.2004.01.018
6. Baldinelli, G. (2009). Double skin façades for warm climate regions: Analysis of a solution with an integrated movable shading system. *Building and Environment, 44*, 1107–1118. https://doi.org/10.1016/j.buildenv.2008.08.005
7. Bessoudo, M., Athienitis, A. K., & Zmeureanu, R. (2010). Indoor thermal environmental conditions near glazed facades with shading devices – Part II: Thermal comfort simulation and

impact of glazing and shading properties. *Building and Environment, 45*, 2517–2525. https://doi.org/10.1016/j.buildenv.2010.05.014
8. Nizovtsev, M. I., Letushko, V. N., Borodulin, V. Yu., & Sterlyagov, A. N. (2020). Experimental studies of the thermo and humidity state of a new building facade insulation system based on panels with ventilated channels. *Energy and Buildings, 206*, 109607. https://doi.org/10.1016/j.enbuild.2019.109607
9. Gagliano, A., & Aneli, S. (2020). Analysis of the energy performance of an opaque ventilated façade under winter and summer weather conditions. *Solar Energy, 205*, 531–544. https://doi.org/10.1016/j.solener.2020.05.078
10. Zhangabay, N., Baidilla, I., Tagybayev, A., & Sultan, B. (2023). Analysis of thermal resistance of developed energy-saving external enclosing structures with air gaps and horizontal channels. *Buildings, 13*(2), 356. https://doi.org/10.3390/buildings13020356
11. Zhangabay, N., Bonopera, M., Baidilla, I., Utelbayeva, A., & Tursunkululy, T. (2023). Research of heat tolerance and moisture conditions of new worked-out face structures with complete gap spacings. *Buildings, 13*(11), 2853. https://doi.org/10.3390/buildings13112853
12. Zhangabay, N., Tursunkululy, T., Ibraimova, U., & Abdikerova, U. (2024). Energy-efficient adaptive dynamic building facades: A review of their energy efficiency and operating loads. *Applied Sciences, 14*(23), 10979. https://doi.org/10.3390/app142310979
13. Lykov, A. V. (1978). *Teplomassobmen: Spravochnik* [Heat and mass transfer: Handbook]. Moscow: Energiya. https://djvu.online/file/Qv0jGOssnL3dk *(in Russian)*
14. Kutateladze, S. S. (1990). *Teploperedacha i gidrodinamicheskoe soprotivlenie: Spravochnoe posobie* [Heat transfer and hydrodynamic resistance: Reference manual]. Moscow: Energoatomizdat. https://search.rsl.ru/ru/record/01001514242 *(in Russian)*
15. Isachenko, V. P., Osipova, V. A., & Sukomel, A. S. (1975). *Teploperedacha: Uchebnik dlya vuzov* (3rd ed., revised and expanded) [Heat transfer: University textbook]. Moscow: Energiya. https://djvu.online/file/BXpZJMYm45EsC *(in Russian)*