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RESEARCH ARTICLE

STRENGTHENING OF STRENGTH OF REINFORCED CONCRETE RAILWAY SLEEPERS UNDER DYNAMIC EFFECTS

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Abstract. *In order to increase the resistance of sleepers to dynamic effects and to strengthen their load-bearing capacity, it is important to investigate their internal strength reserve to various external effects, including the ductility of the subgrade. This is due to the insufficient theoretical and experimental studies to predict their strength reserve in modern conditions. The aim of this research is to improve the dynamic calculation of railway sleepers by developing more accurate calculation models and using modern algorithms to determine their bearing capacity. To achieve this goal, the paper uses analytical and numerical methods based on mathematical models. The base of sleepers is continuous elastic. Methods for determining the natural frequencies of transverse vibrations and a dynamic calculation method for determining the stress-strain state at different load speeds and base stiffness are presented. An example of calculation of railway sleepers is presented. The values of natural frequencies for different forms of vibrations are determined by analytical and numerical methods, and the results are compared. Static and dynamic calculations were carried out, as a result of which the values of internal forces, values of deflections and normal stresses were obtained. Evaluation of the obtained results is given. Strength reserve has been determined and appropriate conclusions have been drawn.*

Keywords: *railway sleepers, elastic base, natural frequencies, stresses, deformations.*

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ҒЫЛЫМИ МАҚАЛА

ДИНАМИКАЛЫҚ ӘСЕРІНЕН ТЕМІР ЖОЛДАРДЫҢ ТЕМІРБЕТОН ШПАЛДАРЫНЫҢ БЕРІКТІГІ КҮШЕЙТУ

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Аңдатпа. *Шпалдардың динамикалық әсерге төзімділігін арттыру және олардың көтергіштігін арттыру үшін олардың беріктігінің ішкі резервін әртүрлі сыртқы әсерлерге, соның ішінде негіздің беріктігіне зерттеу маңызды. Бұл қазіргі жағдайда олардың беріктік резервін болжау үшін жеткілікті теориялық және эксперименттік зерттеулерден туындамайды. Бұл зерттеулердің мақсаты дәлірек есептеу модельдерін жасау және олардың жүк көтергіштігін анықтау үшін заманауи алгоритмдерді қолдану арқылы теміржол байланыстарының динамикалық есебін жетілдіру болып табылады. Осы мақсатқа жету үшін жұмыста математикалық модельдерге негізделген аналитикалық және сандық әдістер қолданылады. Шпалдардың негізі қатты серпімді. Көлденең ауытқулардың нақты жиіліктерін анықтау және жүктеме мен негіздің қаттылығының әртүрлі жылдамдықтарындағы кернеулі деформацияланған күйді анықтау үшін динамикалық есептеу әдістері ұсынылған. Теміржол байланыстарын есептеу мысалы келтірілген. Аналитикалық және сандық әдістердің көмегімен ауытқудың әртүрлі формалары үшін нақты жиіліктердің мәндері анықталды, сонымен қатар нәтижелерді салыстыру жүргізілді. Статикалық және динамикалық есептеулер жүргізілді, нәтижесінде ішкі күштердің мәндері, иілу мәндері және қалыпты кернеулер алынды. Алынған нәтижелер бағаланды. Беріктік резерві анықталды, тиісті қорытындылар жасалды.*

Түйін сөздер: *теміржол шпалдары, серпінді негізі, меншікті жиілігі, кернеу, деформация*

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НАУЧНАЯ СТАТЬЯ

УСИЛЕНИЕ ПРОЧНОСТИ ЖЕЛЕЗОБЕТОННЫХ ШПАЛ ЖЕЛЕЗНЫХ ДОРОГ ПРИ ДИНАМИЧЕСКИХ ВОЗДЕЙСТВИЯХ

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Аннотация. *Для повышения сопротивления шпал динамическим воздействием и усиления их несущей способности важным является изучение внутреннего резерва их прочности на различные внешние воздействия, включая прочность основания. Это вызвано недостаточно теоретическими и экспериментальными исследованиями для прогнозирования их резерва прочности в современных условиях. Целью данных исследований является совершенствование динамического учета железнодорожных шпал путем использования современных алгоритмов для разработки более точных вычислительных моделей и определения их несущей способности. Для достижения этой цели в работе используются аналитические и количественные методы, которые основаны на математических моделях. Основание шпал является сплошным упругим. Предложены методы определения удельных частот поперечных отклонений и динамического расчета по определению напряженно-деформированного состояния при различных скоростях движения нагрузки и жесткости основания. Представлен пример расчета железнодорожных шпал. С помощью аналитических и количественных методов определены значения удельных частот для различных форм отклонений, а также было проведено сравнение результатов. Были проведены статические и динамические расчеты, в результате которых были получены значения внутренних сил, значения изгибов и нормальных напряжений. Полученные результаты были оценены. Определен резерв прочности, сделаны соответствующие выводы.*

Ключевые слова: *железнодорожные шпалы, упругое основание, частота колебаний, напряжение, деформация.*

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Авторы заявляют, что конфликта интересов нет.

1 INTRODUCTION

An important aspect of the development of railway tracks and their improvement is traffic safety, which depends on the quality of the track, its strength, stability and durability. One of the important elements of the reliability of the railway track are reinforced concrete sleepers, which ensure the stability of the rails, reducing the likelihood of subsidence when moving trains.

A railway track is a complex open technical system that interacts with its environment - other transport and non-transport technical systems and the natural environment.The design of a railway track must have coordinated physical and mechanical parameters of its constituent elements and, when a train is moving, accumulate a minimum of potential energy and dissipate this energy in a uniform flow through deformation of materials and relative displacements of the design elements. Sleepers play a key role in ensuring the operational characteristics of the track and safety in rail transport. They are designed to distribute the load from passing trains on the ballast, as well as to hold the rail strings in place, preventing their shift or movement. A reinforced concrete sleeper is a solid bar structure made of reinforced concrete with high-strength reinforcement, which must meet all the requirements of the standard **(Figure 1)**.

In the conditions of increasing the carrying capacity and intensity of freight turnover on the railways of the Republic of Kazakhstan, it is necessary to increase the requirements for traffic safety, one of the areas for this is the study of the strength and reliability of reinforced concrete sleepers capable of withstanding dynamic impacts.

Let us note some theoretical and experimental studies of the strength and reliability of railway sleepers, which reflect monitoring and innovative developments according to research by **Mirsayapov (2022)**. For example, methods for calculating the endurance and crack resistance of reinforced concrete sleepers with prestressed rod reinforcement under cyclic loads.

The studies by **Gnezdilov, Lebedev, and Prostakov (2023)** demonstrate that an innovative diagnostic complex is used to determine and evaluate the stress–strain state of reinforced concrete sleepers on sections of heavy-duty trains under dynamic loads. A review of materials for reinforced concrete sleepers is given, and the main types of failure of prestressed concrete sleepers under cyclic and impact loads are indicated.

The studies by **Mirzakhidova (2021)** present the design features of reinforced concrete sleepers depending on the types of rail fastening to the sleeper, the type of tensioned reinforcement, the presence of electrical insulating properties, and the overall quality of manufacture. The COSMOS/M software system was used for the calculations.

Despite the fact that theoretical and diagnostic studies of the stress-strain state of reinforced concrete sleepers are constantly being conducted, the available data are insufficient to predict their strength reserve under modern conditions. For example, one of the disadvantages of reinforced concrete sleepers is their sensitivity to impacts (especially at the joints), and their insufficient rigidity contributes to rapid wear of the rails at the joints. At the same time, the increased rigidity of reinforced concrete sleepers can contribute to an increase in the dynamic impact of rolling stock on the ballast and on the roadbed, which leads to a more intensive accumulation of track settlements and an increase in the volume of alignment work, especially at the joints. The peculiarity of the sleepers' operation under dynamic influences and their neglect shows numerous defects in the sleepers, which leads to increased labor costs. For effective operation of sleepers, it is necessary to develop clear criteria for their use, a systematic approach to research and the use of accurate mathematical methods.

To increase the resistance of sleepers to dynamic impact and to enhance their bearing capacity, it is necessary to study their internal reserve of strength to various external impacts, including the flexibility of the base as considered in the work of **Leontiev (2020)**.

2 LITERATURE REVIEW

The purpose of these studies is to improve the dynamic calculation of railroad sleepers by developing more accurate calculation models and using modern algorithms to enhance their bearing capacity.

To achieve this goal, analytical and numerical methods are used, based on mathematical models of sleepers, verified by experimental data.

In engineering practice, beam elements of structures lying on a continuous elastic foundation are often encountered. Such structures may include railroad sleepers, strip foundations of buildings, dam foundations resting on soil, various types of pipelines laid on or inside the soil, etc.

In the following, a beam with an uneven cross-section lying on an elastic foundation is adopted as the design scheme for railway sleepers. Concentrated dynamic loads varying according to a harmonic law with frequency θ are considered as external influences, as described by **Mirsayapov (2022) (Figure 1)**.

Figure 1 – Railway sleepers **(Mirsayapov, 2022)**

When calculating sleepers, it is assumed that the soil has elastic properties and its deformation is proportional to the applied load. In addition to this basic premise, other assumptions are also made when calculating beams on an elastic foundation:

- there is no friction between the base and the beam;
- the elastic base is uniform along the entire length of the beam and the width of the beam bed is constant;

One of the most common hypotheses is the hypothesis of a proportional relationship between reaction and settlement—the Winkler foundation hypothesis. The calculation scheme of a beam on an elastic foundation is shown in **Figure 2**, as presented by **Gnezdilov, Lebedev, and Prostakov (2023)**:

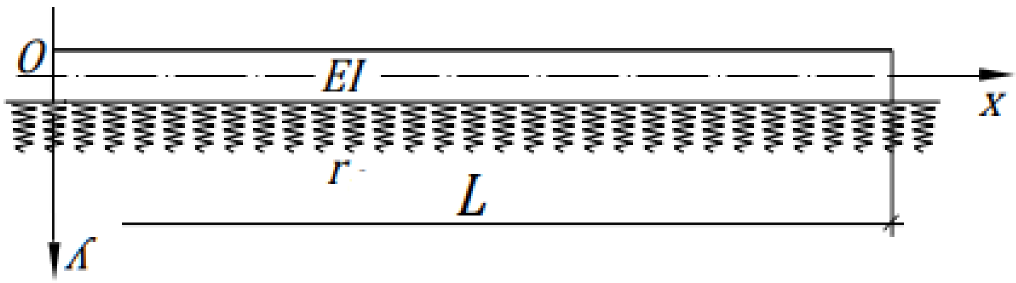

Figure 2 – Calculation scheme of a beam on an elastic foundation (Gnezdilov S.A., 2023).

3 MATERIALS AND METHODS

The research methods consist of 2 parts: the first part is devoted to determining the dynamic characteristics of the system under consideration, and the second part is devoted to the dynamic calculation of railway sleepers, as a result of which their bearing capacity and deformability at high train speeds are clarified.

3.1 Definition of dynamic characteristics

Let us consider the natural oscillations of a beam with a constant mass lying on an elastic foundation. Assuming the external load to be zero, the equation of natural oscillations takes the form as presented by **Dostanova, Issakhanov, Tokpanova, and Kasymova (2024)**:

$$\mathrm{EI}\frac{\partial^4 y(x,t)}{\partial x^4} + m\frac{\partial^2 y(x,t)}{\partial t^2} + rby(x,t) = 0 \tag{1}$$

In (1) y(x,t) is the deflection function, EI is the bending rigidity, m is the linear mass, r is the elastic foundation rigidity. b is the beam width.

Considering periodic oscillations of a beam with a frequency ω, then the deflections can be represented as the following sum of natural oscillations:

$$y(x,t) = \sum_{k=1}^{\infty} y_k(x)\, T_k(t) \quad \text{or}$$

$$y(x,t) = \sum_{k=1}^{\infty} y_k(x)\, Sin\omega_k t \tag{2}$$

Substituting (2) into (1), reducing by $\sin\omega t$ for the “k” mode of vibrations, we obtain the following equation

$$\mathrm{EI}\frac{\partial^4 y_k(x)}{\partial x^4} - \left[\frac{m\omega_k^2 - rb}{\mathrm{EI}}\right] y_k(x) = 0 \tag{3}$$

In the following we will denote the frequency parameter

$$\lambda_k = \sqrt[4]{\left[\frac{m\omega_k^2 - rb}{\mathrm{EI}}\right]}$$

The boundary conditions are as follows:

$$x=0 \quad M(0)=Q(0)=0 \tag{3}$$
$$x=L \quad M(L)=Q(L)=0 \tag{4}$$

We use the initial parameter method to solve equation (3), assuming two initial parameters $M_0=Q_0=0$, then the solution to the homogeneous equation can be written as:

$$\begin{aligned} y_k^0(x) &= y_0 S(\lambda_k x) + \frac{\varphi_0}{\lambda_k} T(\lambda_k x) \\ \varphi_k^0(x) &= y_0 \lambda_k V(\lambda_k x) + \varphi_0 S(\lambda_k x) \\ M_k^0(x) &= -\mathrm{EI} y_0 \lambda_k^2 U(\lambda_k x) - \mathrm{EI}\varphi_0 \lambda_k V(\lambda_k x) \\ Q_k^0(x) &= -\mathrm{EI} y_0 \lambda_k^3 T(\lambda_k x) - \mathrm{EI}\varphi_0 \lambda_k^2 U(\lambda_k x) \end{aligned} \tag{5}$$

k=1,2,3………

In (5) the following circular functions are introduced

$$S(\lambda_k x) = \frac{1}{2}(ch\lambda_k x + cos\lambda_k x)$$
$$U(\lambda_k x) = \frac{1}{2}(ch\lambda_k x - cos\lambda_k x)$$
$$T(\lambda_k x) = \frac{1}{2}(sh\lambda_k x + sin\lambda_k x)$$

$$V(\lambda_k x) = \frac{1}{2}(sh\lambda_k x - sin\lambda_k x) \quad (6)$$

Unknown initial parameters deflection y_0 and rotation angle φ_0 are determined from the boundary conditions (3,4):

$$M_k^0(\lambda_k L) = -EIy_0\lambda_k^2\, U(\lambda_k L) - EI\varphi_0\lambda_k V(\lambda_k L) = 0$$
$$Q_k^0(\lambda_k L) = -EIy_0\lambda_k^3 T(\lambda_k L) - EI\varphi_0\lambda_k^2 U(\lambda_k L)=0 \quad (7)$$

Equations (7) represent a system of two homogeneous algebraic equations with respect to two unknown initial parameters (y_0, φ_0), since the solution must be different from zero, we assume that the determinant of the system is zero:

$$D = U^2(\lambda_k L) - V(\lambda_k L)\cdot T(\lambda_k L) = 0 \quad (8)$$

Equation (8) is a transcendental equation with respect to the parameter λk. The solution can be obtained by an approximate method.

To solve (8), we use an iterative process for the parameter λ in the range of ω [π/2 - 2π]. We introduce the function

$$F(\lambda_k) = U^2(\lambda_k L) - V(\lambda_k L)\cdot T(\lambda_k L) = 0 \quad (9)$$

Let us designate

$$A = U^2(\lambda_k L) \quad and \quad B = V(\lambda_k L)\cdot T(\lambda_k L)$$

Then equation (8) can be replaced by the equation:

A-B=0 or A=B

By specifying different values of λ_k, the values of A and B are determined. This process can be linearized. The iterative process continues until the value of the function reaches the required accuracy $F(\lambda_k)=A-B<\varepsilon$, where ε is a sufficiently small number characterizing the required accuracy of the calculation.

Having determined λ_k, the natural frequency of oscillations is determined by the formula:

$$\omega_k = \sqrt[4]{\left[\frac{EI\lambda_k^4 + rb}{m}\right]} \quad (10)$$

k=1,2,3…………

3.2 Dynamic calculation

Let us consider the forced vibrations of a beam with a constant mass. The equation of forced vibrations takes the form as presented by **Pichkurova (2025)**:

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + m\frac{\partial^2 y(x,t)}{\partial t^2} + rby(x,t) = q(x,t) \quad (11)$$

If the external load changes according to the harmonic law with frequency θ, i.e. $y(x,t)=sin\theta t$, then it can be assumed that the beam also oscillates according to the harmonic law with the same frequency θ. The solution of equation (11) can be expanded as a series and presented as the following sum:

$$y(x,t) = \sum_{k=1}^{\infty} y_k(x) sin\theta_k t \quad (12)$$

Substituting (12) into (11), reducing by sinθt for the "k" mode of oscillations, we obtain the following equation

$$EI\frac{\partial^4 y(x)}{\partial x^4} - \left[\frac{m\theta_k^2 - rb}{EI}\right] y_k(x) = q_0(x) \tag{13}$$

In (13) q_0 is the amplitude value of the external load. In the following, by analogy with natural oscillations, we denote

$$\lambda_k = \sqrt[4]{\left[\frac{m\theta_k^2 - rb}{EI}\right]} \tag{14}$$

Equation (13) is an ordinary differential equation. Solution (13) can be represented as the sum of the solution of the homogeneous equation and a particular solution:

$$y_k(x) = y_k^o(x) + y_k^{\text{част}}(x) \tag{15}$$

When considering forced vibrations of beamswith a uniformly distributed mass m, we consider the case when a concentrated external moving load F changes according to a harmonic law with a frequency θ (for the case of an arbitrary load, we use the expansion according to the norms of natural vibrations):

$$F = F_0 sin(\frac{\pi t}{T_0}), \tag{16}$$

where F_0 is the wheel load distributed over the area of a circle of diameter D, $T_0 = \frac{D}{V}$, V is the speed of horizontal movement of the load, t is the current time. Denoting the ratio π/T_0 through θ, then the external load can be represented as $F = F_0 sin(\theta \cdot t)$, where θ is the frequency of oscillations of the external load.

In the case of the action of two concentrated forces F (Figure 3), the solutions for the two sections are given as presented by **Dostanova, Shayakhmetov, Lesov, Umarov, and Kalpenova (2025)**:

At $0 \leq x \leq a$

$$\begin{gathered} y_k(x) = y_0 S(\lambda_k x) + \frac{\varphi_0}{\lambda_k} T(\lambda_k x) \\ \varphi_k(x) = y_0 \lambda_k V(\lambda_k x) + \varphi_0 S(\lambda_k x) \\ M_k(x) = -EIy_0 \lambda_k^2 U(\lambda_k x) - EI\varphi_0 \lambda_k V(\lambda_k x) \\ Q_k(x) = -EIy_0 \lambda_k^3 T(\lambda_k x) - EI\varphi_0 \lambda_k^2 U(\lambda_k x) \\ k=1,2,3\ldots\ldots\ldots \end{gathered} \tag{17}$$

At $a \leq x \leq L$

$$\begin{gathered} y_k(x) = y_0 S(\lambda_k x) + \frac{\varphi_0}{\lambda_k} T(\lambda_k x) + \frac{FV(\lambda(x-a))}{\lambda^3 EI} \\ \varphi_k(x) = y_0 \lambda_k V(\lambda_k x) + \varphi_0 S(\lambda_k x) + \frac{FU(\lambda(x-a))}{\lambda^2 EI} \\ M_k(x) = -EIy_0 \lambda_k^2 U(\lambda_k x) - EI\varphi_0 \lambda_k V(\lambda_k x) - \frac{FT(\lambda(x-a))}{\lambda} \\ Q_k(x) = -EIy_0 \lambda_k^3 T(\lambda_k x) - EI\varphi_0 \lambda_k^2 U(\lambda_k x) - FS(\lambda(x-a)) \end{gathered} \tag{18}$$

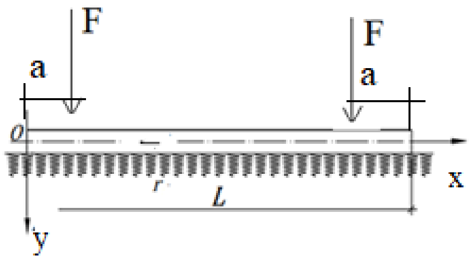

Figure 3 – The action of concentrated external forces **(Dostanova S., Shayakhmetov S., 2025)**

In (17) and (18) the parameter$\lambda_k$is expressed as:

$$\lambda_k = \sqrt[4]{\left[\frac{m\theta_k^2 - rb}{EI}\right]}$$

Initial parameters y_0 and φ_0 under forced oscillations are determined from the boundary conditions:

$$x=L \quad M(L)=Q(L)=0$$

$$-EIy_0\lambda_k^2 U(\lambda_k L) - EI\varphi_0\lambda_k V(\lambda_k L) = \frac{FT(\lambda(L-a))}{\lambda} \tag{19}$$

$$-EIy_0\lambda_k^3 T(\lambda_k L) - EI\varphi_0\lambda_k^2 U(\lambda_k L) = FS(\lambda(L-a))$$

Due to symmetry, equations (19) have the form:

At $\quad a \le x \le L$

$$x=L/2 \quad \varphi(L/2)=0, \quad Q(L/2)=0$$

$$\varphi_k(L/2) = y_0\lambda_k V(\lambda_k L/2) + \varphi_0 S(\lambda_k L/2) + \frac{FU(\lambda(L/2-a))}{\lambda^2 EI} = 0$$

$$Q_k(L/2) = -EIy_0\lambda_k^3 T(\lambda_k L/2) - EI\varphi_0\lambda_k^2 U\left(\frac{\lambda_k L}{2}\right) - FS\left(\lambda\left(\frac{L}{2} - a\right)\right) = 0 \tag{20}$$

Having determined the initial parameters from (20), the deflections, rotation angles, bending moment and shear force are determined from expressions (17,18).

4 RESULTS AND DISCUUION

The most common reinforced concrete sleepers on the railway in the Republic of Kazakhstan—type Sh1-1, used for high-speed lines—were selected for research, as presented by **Dostanova, Kasymova, Tokpanova, Tulegenova, and Sarsenova (2024)**.

Characteristics of reinforced concrete sleepers type Ш1-1: track width - 1520 mm, dimensions: length - 2700 mm; height - 150 mm; width - 300 mm, material - prestressed reinforced concrete **(Figure 4)**.

Concrete compressive strength class – B40 (M500). E=36x10^3MPa, specific gravity of concrete γ=2.5 t/m^3. Distance between stop edges of different ends of sleeper – L=2016 mm, stop edge angle – 55^0, sleeper sub-section height – 193 mm, sleeper average section height – h=145 mm, fastening type KB-65.

The train speeds are considered in the range from 60 to 140 km/h (Figure 4), as presented by **Dostanova, Shayakhmetov, Lesov, Umarov, and Kalpenova (2025)**.

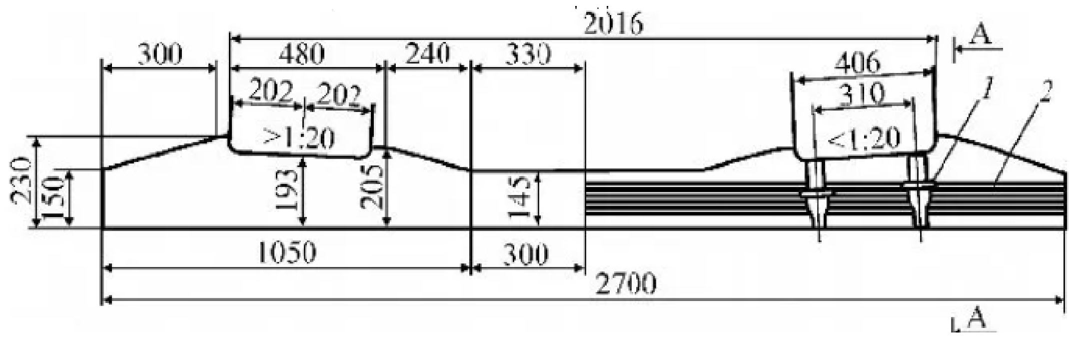

Figure 4 – Reinforced concrete sleeper type Sh1-1 **(Dostanova S., Shayakhmetov S., 2025)**

Rigidity of the base or values of the sleeper bedding coefficient, MN/m^3:
for soil base without ballast: 8... 12
for sand ballasts: 15...40
for gravel ballasts: 40...60
for crushed stone ballast: 60... 100

In the following, we consider crushed stone ballast with a sleeper bedding coefficient in the range from 60 to 80 MN/m^3. In the following, we present the results for r=60 MN/m^3.

Figure 5 shows the numbers of the sections and points at which deflections, bending moments, shear forces, and the corresponding stresses are determined, as presented by **Dostanova, Shayakhmetov, Lesov, Umarov, and Tokpanova (2025)**.

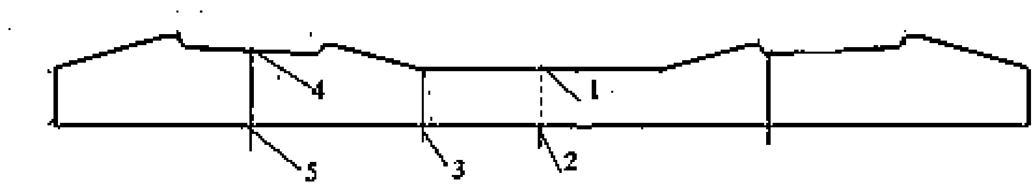

Figure 5 – Nnumbers of sections and points at which forces and deformations are determine **(Dostanova S., Shayakhmetov S., 2025)**

The moment of inertia of section 4-5 is: I=30417.5 cm^4; section 1-2: I=7621.5 cm^4, in section 3: I=8437.5 cm^4, respectively, the moments of resistance of the sections are: section 4-5 is: W=2645.5 cm^3; section 1-2: W=1051.2 cm^3 in section 3: W=1125.0 cm^3. The calculation is carried out in the elastic stage of the system operation and consists of 2 parts: determination of the dynamic characteristics and determination of the stress-strain state of reinforced concrete sleepers when the train is moving at a given speed.

The natural frequencies of oscillations are determined analytically and numerically using the COSMOS/M software system. In the latter, the STAR module presents the calculation of the stress-strain state and the calculation of the displacements of a reinforced concrete sleeper.

Figure 6 shows the shape of the natural oscillations corresponding to the first harmonic, obtained as a result of numerical calculations, as presented by **Dostanova, Shayakhmetov, Lesov, Umarov, and Tokpanova (2025)**.

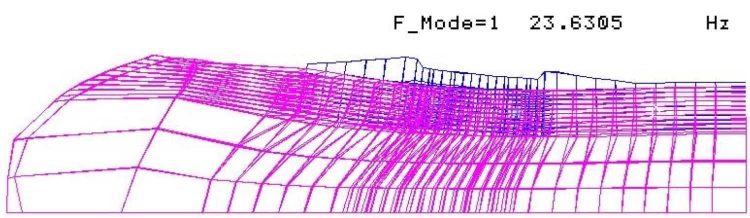

Figure 6 – The shape of natural oscillations corresponding to the 1st harmonic **(Dostanova S., Shayakhmetov S., 2025)**

Table 1 shows the natural frequencies of oscillations of the Ш1 sleeper, ω Hz, corresponding to the first harmonics.

Table 1
Natural frequencies of oscillations

Form No.	Analytical calculation	Numerical calculation	Errors in %
1	22,51	23.63	4,7
2	30,77	31.8	3,2
3	38,72	40.48	4,3
4	41,47	42.77	3,0
5	47,35	46.97	0,8

From **Table 1** it is evident that the natural frequencies determined analytically have lower values in comparison with the numerical method; with an increase in the vibration modes, their values almost coincide.

The results of forced vibrations are presented at the speed of movement V=60 km/h. The value $\lambda_к=\sqrt[4]{\frac{rb}{EI}}$ =0,0636 ((section 1-2), for section 4-5: λ_k=0.002, for section 3: λ_k=0.062. The functions S(λx), T (λx), V (λx), U(λx) are determined by formula (6) using tables for trigonometric and hyperbolic functions. **Table 2** shows the values of deflections, bending moments and transverse forces under static and dynamic action of force F ($F_{ст}$ =270 kN, F_{din}=270sinθt, where θ is the frequency of forced vibrations, at the speed of movement of 60 km/h θ=65.41 sec^{-1}).

Table 2
Deflections m, bending moments kNm and transverse forces kN in sections of reinforced concrete sleepers under static and dynamic effects.

Section number	Deflections, 10^{-3} m	Bending moments kNm	Transverse forces kN
		Static impact	
1-2	10,91	19,30	0
4-5	12,15	-28,59	100,98 -14,18
3	11,72	5,48	-95,62
		Dynamic impact	
1-2	13,74	26,24	0
4-5	16,52	-38,88	137,33 -19,28
3	15,93	7,45	-130,04

From **Table 2** it can be seen that the dynamic coefficient is approximately μ_{din}=1,36.

Table 3 shows the values of maximum normal and shear stresses (Ϭx, kgf/cm^2, τ_{yx},, kgf/cm^2) in the sections of reinforced concrete sleepers.

Table 3
Values of maximum normal stresses Ϭx, kgf/cm^2

Number sections	Ϭx, kgf/cm^2 Statics	τ_{yx}, kgf/cm^2 Statics	Ϭx, kgf/cm^2 Dynamics	τ_{yx}, kgf/cm^2 Dynamics
1-2	74,43	0	101,22	0
4-5	277,41	22,4 -3,14	377,28	30,5 -4,3
3	49,69	23,8	67,78	32,4

According to the building codes of the Republic of Kazakhstan, the average compressive strength of B40 concrete, taking into account the variation coefficient of 13.5%, is 523.7 kgf/cm^2, bending strength is 349 kgf/cm^2, tensile strength is 48 kgf/cm2, and shear strength varies in the range from 10.2 to 61.2 kgf/cm^2.

Table 3 shows that the highest normal stresses are in section 4-5, the deflections in this section are also increased in comparison with the section in the middle of the sleeper span, and with an increase in load, cracks may appear on the outer surface. The safety factor for normal stresses is 1.39. The highest tangential stresses are in section 3, in this section, shifts in the direction of the y axis are possible, which can lead to an emergency situation when trains are moving. It should be noted that when calculating railway sleepers, shear deformation is often neglected, considering it small in comparison with bending deformation. As the results of this calculation show, the safety reserve for shear is quite small.

5 CONCLUSIONS

Summarizing the obtained results, the following conclusions can be drawn:

1. Using analytical methods, it is possible to identify and specify the dynamic reserve of strength and reliability of railway scales of railways at different speeds of movement and at different values of base rigidity. These results will allow predicting the behavior of sleepers at different values of weight and speed of the moving train.

2. Practice shows that failure to take into account dynamic effects when calculating railroad sleepers leads to the appearance of various defects. To enhance the bearing capacity and reliability, it is necessary to take into account all possible types of deformations and their stability.

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