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RESEARCH ARTICLE

DYNAMIC CHARACTERISTICS FOR TRANSPORT STRUCTURES WITH ELASTIC SUPPORTS

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Abstract. *Currently, the issues of strengthening the load-bearing capacity of transport structures under the action of dynamic loads are becoming increasingly relevant. To ensure a reserve of structural strength under dynamic impacts, active methods of protection are increasingly being used. The use of active methods of protection in the form of pliable supports makes it possible to increase the energy intensity of the "support-structure" system and reduce the intensity of the dynamic impact. They play the role of dampers, reducing vibration amplitudes and absorbing deformation energy. The dynamic characteristics of transport structures play a significant role in the oscillatory process, because the resistance of the structure to external influences depends on them. By changing them, you can lower the value of the dynamic coefficient, thereby reducing the dynamic effect. The aim of the research is to evaluate the influence of the stiffness of the support links of transport structures on the dynamic characteristics of road overpass spans. To determine the natural frequencies, a homogeneous partial differential equation is used, the solution of which is presented using hyperbolic functions. The solution of the transcendental equation is obtained using the approximation method. The relevance of the obtained results lies in the fact that when using elastically supple supports in transport structures it is possible to change dynamic characteristics and thereby increase the strength reserve and reduce the dynamic effect.*

Keywords: *compliance, rigidity of supports, natural frequencies, inertial forces, approximation method..*

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ҒЫЛЫМИ МАҚАЛА

СЕРПІМДІИКЕМДЕЛГЕН ТІРЕКТЕРІ БАР КӨЛІК ҚҰРЫЛЫМДАРЫНЫҢ ДИНАМИКАЛЫҚ СИПАТТАМАЛАРЫ

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Аңдатпа. *Қазіргі уақытта динамикалық жүктемелердің әсерінен көліктік конструкциялардың жүк көтергіштігін күшейту мәселелері өзекті болуда. Динамикалық әсерлер кезінде конструкциялық беріктік қорын қамтамасыз ету үшін қорғаныстың белсенді әдістері көбірек қолданылуда. Берілгіш тіректер түріндегі қорғаныстың белсенді әдістерін қолдану «тірек-конструкция» жүйесінің энергия сыйымдылығын арттыруға және динамикалық әсердің қарқындылығын төмендетуге мүмкіндік береді. Олар тербеліс амплитудаларын төмендететін және деформация энергиясын сіңіретін демпферлердің рөлін атқарады. Көлік конструкцияларының динамикалық сипаттамалары тербелмелі процесте маңызды рөл атқарады, себебі конструкцияның сыртқы әсерлерге төзімділігі соларға байланысты. Оларды өзгерту арқылы динамикалық коэффициенттің мәнін төмендете отырып, динамикалық әсерді азайтуға болады. Бұл жұмыста тірек байланыстарының қаттылығына байланысты автомобиль өтпе аралықтарының меншікті тербеліс жиіліктерінің өзгеруі зерттеледі. Алынған нәтижелердің өзектілігі көлік конструкцияларында серпінді тіректерді пайдалану кезінде серпінді сипаттамаларды өзгертуге және осылайша беріктік резервін арттыруға және серпінді әсерді төмендетуге болатындығында.*

Түйін сөздер: *икемделгіштік, тіректердің қаттылығы, меншікті жиіліктер, инерциялық күштер, жуықтау әдісі.*

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НАУЧНАЯ СТАТЬЯ

ДИНАМИЧЕСКИЕ ХАРАКТЕРИСТИКИ ДЛЯ ТРАНСПОРТНЫХ КОНСТРУКЦИЙ С УПРУГОПОДАТЛИВЫМИ ОПОРАМИ

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Аннотация. *В настоящее время большую актуальность приобретают вопросы усиления несущей способности транспортных конструкций при действии динамических нагрузок. Для обеспечения резерва прочности конструкций при динамических воздействиях все более используют активные способы защиты. Применение активных способов защиты в виде податливых опор позволяет повысить энергоемкость системы «опора-конструкция» и снизить интенсивность динамического воздействия. Они играют роль демпферов, снижая амплитуды колебаний и поглощая энергию деформаций. Динамические характеристики транспортных сооружений играют значительную роль в колебательном процессе, т.к. от них зависит сопротивление конструкции внешним воздействиям. Изменяя их, можно понизить значение динамического коэффициента, тем самым снизить динамический эффект. Целью исследований является оценка влияния жесткости опорных связей транспортных конструкций на динамические характеристики пролетных строений автодорожных путепроводов. Для определения собственных частот используется однородное дифференциальное уравнение в частных производных, решение которого представлено с использованием гиперболических функций. Решение трансцендентного уравнения получено с использованием метода приближений. Актуальность полученных результатов заключается в том, что при использовании упругоподатливых опор в транспортных конструкциях можно изменять динамические характеристики и тем самым повысить резерв прочности и снизить динамический эффект.*

Ключевые слова: *податливость, жесткость опор, собственные частоты, инерционные силы, метод приближений*

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CONFLICT OF INTEREST

The authors state that there is no conflict of interest.

АЛҒЫС/ҚАРЖЫЛАНДЫРУ КӨЗІ

Зерттеу жеке қаржыландыру көздерін пайдалана отырып жүргізілді.

МҮДДЕЛЕР ҚАҚТЫҒЫСЫ

Авторлар мүдделер қақтығысы жоқ деп мәлімдейді.

БЛАГОДАРНОСТИ/ИСТОЧНИК ФИНАНСИРОВАНИЯ

Исследование проводилось с использованием частных источников финансирования.

КОНФЛИКТ ИНТЕРЕСОВ

Авторы заявляют, что конфликта интересов нет.

1 INTRODUCTION

The analysis of oscillatory processes of transport structures, especially road bridges, under the influence of mobile load is becoming increasingly relevant. As the weight and intensity of the moving load increases with the length of the overlapping spans, dynamic phenomena increase and this causes increased requirements for their operation.

In the process of vibrations, the ability of a structure to resist external influences depends on its dynamic characteristics. Therefore, it is important to strengthen the load-bearing capacity of structures at the expense of internal and external reserves. Traditional methods and means of protecting buildings and structures from dynamic impacts include a large range of various measures aimed at increasing the load-bearing capacity of building structures.

Currently, to ensure the resistance of structures under intense dynamic impacts, active protection methods are increasingly being used. The use of active protection methods in the form of flexible supports allows increasing the energy intensity of the support-structure system and reducing the intensity of dynamic impact. As a result, the cost of structures and the complexity of their restoration is reduced.

The development and implementation of active methods for protecting reinforced concrete structures under dynamic loading requires the development and improvement of effective methods for dynamic calculation of systems that include both the structures themselves and the means of active protection. At the same time, for a reliable assessment of the stress-strain state, the developed calculation methods should take into account not only the basic physical laws of deformation of reinforced concrete under dynamic influences, but also the features of deformation of malleable supports. They absorb strain energy and act as dampers.

The problem of improving methods for calculating reinforced concrete structures on flexible supports under dynamic loading is an actual scientific problem of great practical importance **(Galyautdinov Z.R., 2021; Gridnev S.Yu., 2013; Kolotovichev Yu.A., 2023)**.

Experimental studies show that the use of flexible supports of constant stiffness for loads characterized by the stage of increase and decrease can have both a positive effect on the operation of structures (reduce the amplitudes of vibrations) and a negative one. This circumstance must be taken into account when designing structures on flexible supports in order to avoid the appearance of large forces and displacements in the structures compared to structures on non-displaced supports. To increase the effectiveness of increasing the resistance of reinforced concrete structures to dynamic impacts, it is advisable to use flexible supports of variable stiffness. To do this, it is necessary to optimize the rigidity of flexible supports, taking into account the physical and geometric parameters of spans **(Galyautdinov Z.R., 2021; Gridnev S.Yu., 2013)**.

In the following, the use of elastically malleable supports of superstructures in road overpasses is considered. To optimize the rigidity of flexible supports, it is necessary to deduce the dependences of the natural frequencies of transverse vibrations of superstructures on them, as well as their influence on the stress-strain state of the system under the action of external dynamic loads. Analytical and variation methods are used to solve these problems. It is proposed to vary the stiffness of supports by iteration method when solving the characteristic equation.

2 LITERATURE REVIEW

The development and implementation of active methods for protecting reinforced concrete structures under dynamic loading requires the development and improvement of effective methods for dynamic calculation of systems that include both the structures themselves and the means of active protection. At the same time, for a reliable assessment of the stress-strain state, the developed calculation methods should take into account not only the basic physical laws of deformation of reinforced concrete under dynamic influences, but also the features of deformation of malleable supports. They absorb strain energy and act as dampers.

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The fundamental theory of dynamics and stability of building and transport structures, modern analytical and numerical methods using software packages for calculating dynamic characteristics, the theory of oscillations, and taking into account various types of nonlinearity are considered in many works **(Kadisov G.M., 2012; Yegorychev O.O., 2005; Bekshayev S.Ya., 2013; Sedighi H.M., 2012; Sedighi H.M., 2011; Ryabukhin A.K., 2020; Ray W.Cl., 1977; Korenev B.G., 1972; Nikolaenko N.A., 1998)**.

For buildings and transport structures in seismic areas, issues of seismic protection and seismic isolation, as well as the elimination of resonant vibrations using flexible supports, are important **(Dostanova S.Kh., 2020; 2021)**. При рассмотрении движущейся нагрузки на мостовых конструкциях для уменьшения динамического эффекта необходимо более точно экспериментально и теоретически определять частоты и формы собственных колебаний **(Dostanova S.Kh., 2023)**.

Despite the existing experimental and theoretical studies of transport structures, some issues require clarification when determining dynamic characteristics, searching for an internal reserve of strength, and enhancing reliability and safety under the action of moving loads.

Due to the intensive development of road transport and increasing requirements for their operation, it is necessary to improve and develop experimental and theoretical research using innovations and scientific achievements in the field of road sector.

In the following, the use of elastically malleable supports of superstructures in road overpasses is considered. To optimize the rigidity of flexible supports, it is necessary to deduce the dependences of the natural frequencies of transverse vibrations of superstructures on them, as well as their influence on the stress-strain state of the system under the action of external dynamic loads. Analytical and variation methods are used to solve these problems. It is proposed to vary the stiffness of supports by iteration method when solving the characteristic equation.

3 MATERIALS AND METHODS

3.1 METHOD FOR DETERMINING THE NATURAL VIBRATION FREQUENCIES OF LOAD-BEARING BEAMS OF SUPERSTRUCTURES

Free transverse vibrations of a rod with a distributed mass are considered. The superstructure is considered as a beam on two elastically pliable supports with rigidity C_1 and C_2. It is assumed that each span of the split bridge operates independently of each other. Consider a beam on control supports (**Figure 1**). Free vibrations of a beam with a uniformly distributed mass are described by a fourth-order partial differential equation **(Leontiev E.V., 2020; Korobko V.I., 2007; Bekshayev S.Ya., 2013; Sedighi H.M., 2012; Sedighi H.M., 2011; Ryabukhin A.K., 2020)**.

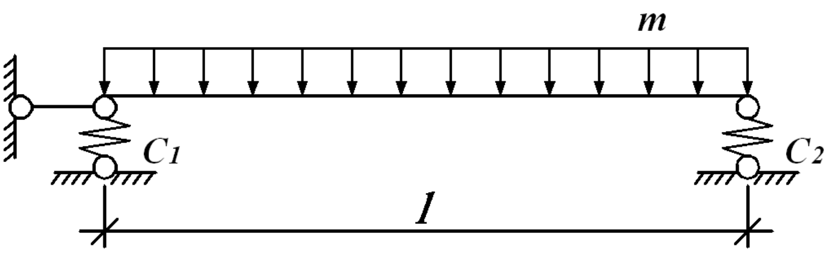

Figure 1 – Design diagram of a beam on regulating supports (authors' material)

The differential equation of free vibrations without taking into account the resistance forces has the form **(Galyautdinov Z.R., 2021; Gridnev S.Yu., 2013; Kolotovichev Yu.A., 2023; Kadisov G.M., 2012; Yegorychev O.O., 2012)**.

$$EI\frac{\partial^4 y}{\partial x^4}+m(x)\frac{\partial^2 y}{\partial t^2}=0 \tag{1}$$

In equation (1) y=y(x,t)- are the unknown deflections, EI - flexural stiffness, m(x)- linear mass. We restrict ourselves to finding only such solutions of equation (1) that determine standing waves, i.e., the bending shape that does not depend on time. These solutions correspond to the main waveforms. For such waveforms, the solution can be obtained by separating the variables, then the solution and equation (1) can be written as:

$$\begin{gathered} y(x,t)=\sum_{k=1}^{\infty} y_k(x,t)=\sum_{k=1}^{\infty} y_k(x)T_k(t), \\ EI(\sum_{k=1}^{\infty} y_k^{(IV)}(x))+m(x)(\sum_{k=1}^{\infty} y_k(x)T_k^{//}(t))=0 \end{gathered} \tag{2}$$

Combining the sums and equating the terms of the same name, we obtain a system of differential equations:

$$EI\frac{\partial^4 y_k(x)}{\partial x^4}T_k(t)+m(x)y_k(x)\frac{\partial^2 T_k(t)}{\partial t^2}=0, \qquad k=1,2,3,....... \tag{3}$$

Dividing each term in (3) by a value $m(x)y_k(x)T_k(t)$, we obtain the sum of two terms, each of which depends on only one variable. We introduce a notation for the relations of the following quantities:

$$\frac{EIy_k^{IV}(x)}{m(x)y_k(x)}=-\frac{T_k^{//}(t)}{T_k(t)}=\omega_k^2 \tag{4}$$

These relations are independent of the variables x,t and represent the squares of the natural vibration frequencies of a beam with a distributed mass. By introducing the notation (4), the system (3) decomposes into two independent systems of ordinary differential equations:

$$y_k^{(IV)}(x)-\lambda_k^4 y_k(x)=0, \quad k=1,2,3,....$$
$$\lambda_k^4=\frac{m(x)\cdot\omega_k^2}{EI} \tag{5}$$

$$T_k''(t)+\omega_k^2 T_k(t)=0, \qquad k=1,2,3.... \tag{6}$$

In equation (6) ωk - the frequency of natural oscillations corresponding to k-form. Using the Euler method for solving linear ordinary equations, the solutions of equations (5) and (6) can be represented as:

$$y(x,t)=\sum_{k=1}^{\infty} y_k(x)T_k(t),$$
$$y_k(x)=Ach(\lambda_k x)+Bsh(\lambda_k x)+C\cos(\lambda_k x)+D\sin(\lambda_k x), \tag{7}$$
$$T_k(t)=A_k\sin\omega_k t+B_k\cos\omega_k t$$

General solution for deflections:

$$y(x,t)=\sum_{k=1}^{\infty}\begin{matrix}(Ach(\lambda_k x)+Bsh(\lambda_k x)+C\cos(\lambda_k x)+D\sin(\lambda_k x))\\(A_k\sin\omega_k t+B_k\cos\omega_k t)\end{matrix},$$

In (7), the values A, B, C, and D are constants determined from the anchoring conditions (boundary conditions), and A_k, B_k a are constants determined from the initial conditions, i.e., displacements and velocities at the initial time t=0.

The expression for $y_k(x)$ defines the main form of vibrations corresponding to the frequency ωk, it defines a static elastic line caused by the running load $q_k=m(x)\omega_k^2 y_k$. If the running mass is constant along the length of the rod, $m(x)=m=const$, then we can write for the parameter s_k :

$$\lambda_k=l\sqrt[4]{\frac{m\omega_k^2}{EI}} \tag{8}$$

For the case when the beam is hinged at the edges, the boundary conditions will be:

$$\text{at } x=0 \qquad M_0=0,\ Q_0=y_0\cdot C_1,$$
$$\text{at } x=l \qquad M_l=0,\ Q_l=-y_l\cdot C_2, \tag{9}$$

where y_0 – displacement at the origin, y_l - beam displacement at $x=l$; M_0 and M_l -moment at $x=0$ and $x=l$, respectively, Q_0 and Q_l -transverse forces at $x=0$ and $x=l$, respectively.

Let us express the coefficients A, B, C, and D in terms of the initial parameters y_0, φ_0, M_0, Q_0, then the standing waves can be represented as **(Leontiev E.V., 2020; Korobko V.I., Korenev B.G., 1972)**:

$$\left.\begin{aligned}
y_x &= y_0 S_x + \frac{\varphi_0}{\lambda} T_x - \frac{M_0}{\lambda^2 EI} U_x - \frac{Q_0}{\lambda^3 EI} V_x, \\
\varphi_x &= y_0 \lambda V_x + \varphi_0 S_x - \frac{M_0}{\lambda EI} - \frac{Q_0}{\lambda^2 EI} U_x, \\
M_x &= -y_0 \lambda^2 EIU_x - \varphi_0 \lambda EIV_x + M_0 S_x + \frac{Q_0}{\lambda} T_x \\
Q_x &= -y_0 \lambda^3 EIT_x - \varphi_0 \lambda^2 EIU_x + M_0 \lambda V_x + Q_0 S_x
\end{aligned}\right\} \tag{10}$$

In (10) S_x, T_x, U_x, V_x are the Krylov functions:

$$S_x = \frac{ch\lambda x + \cos\lambda x}{2}, \qquad T_x = \frac{sh\lambda x + \sin\lambda x}{2},$$
$$U_x = \frac{ch\lambda x - \cos\lambda x}{2}, \qquad V_x = \frac{sh\lambda x - \sin\lambda x}{2}$$

$$\left.\begin{aligned}
y_x &= y_0 S_x + \frac{\varphi_0}{\lambda} T_x - \frac{y_0 C_1}{\lambda^3 EI} V_x \\
\varphi_x &= y_0 \lambda V_x + \varphi_0 S_x - \frac{y_0 C_1}{\lambda^2 EI} U_x, \\
M_x &= -y_0 \lambda^2 EIU_x - \varphi_0 \lambda EIV_x + \frac{y_0 C_1}{\lambda} T_x, \\
Q_x &= -y_0 \lambda^3 EIT_x - \varphi_0 \lambda^2 EIU_x + y_0 C_1 S_x
\end{aligned}\right\} \tag{11}$$

In (11) C_1 - is the stiffness of the support at $x = 0.$ Using (11), we write down the expressions for deflection, moment and transverse force at $x = l.$ As a result we obtain a system of three homogeneous equations with respect to three unknowns: y_0, φ_0, y_l.

$$\left.\begin{aligned}
y_{x=l} &= y_0 S_{x=l} + \frac{\varphi_0}{\lambda} T_{x=l} - \frac{y_0 C_1}{\lambda^3 EI} V_{x=l}, \\
M_{x=l} &= 0 = -y_0 \lambda^2 EIU_{x=l} - \varphi_0 \lambda EIV_{x=l} + \frac{y_0 C_1}{\lambda} T_{x=l} \\
Q_x &= -C_2 y_{x=1} = -y_0 \lambda^3 EIT_{x=l} - \varphi_0 \lambda^2 EIU_{x=l} + y_0 C_1 S_{x=l}
\end{aligned}\right\} \tag{12}$$

$$S_{x=l} = \frac{ch\lambda l + \cos\lambda l}{2}, \qquad T_{x=l} = \frac{sh\lambda l + \sin\lambda l}{2},$$
$$U_{x=l} = \frac{ch\lambda l - \cos\lambda l}{2}, \qquad V_{x=l} = \frac{sh\lambda l - \sin\lambda l}{2}$$

Equating the determinant of the system with unknowns y_0, φ_0, y_i, to zero, we obtain a transcendental equation with respect to the unknown quantity λ.

$$(S_{x=l}-\frac{C_1}{\lambda^3 EI}V_{x=l}-1)y_0+\frac{\varphi_0}{\lambda}T_{x=l}-y_{x=l}=0,$$
$$-y_0(\lambda^2 EIU_{x=l}-\frac{C_1}{\lambda}T_{x=l})-\varphi_0\lambda EIV_{x=l}=0$$
$$-y_0(\lambda^3 EIT_{x=l}-C_1 S_{x=l})-\varphi_0\lambda^2 EIU_{x=l}+C_2 y_{x=l}=0$$

$$\begin{vmatrix} (S_{x=l}-\frac{C_1}{\lambda^3 EI}V_{x=l}-1) & \frac{1}{\lambda}T_{x=l} & -1 \\ -(\lambda^2 EIU_{x=l}-\frac{C_1}{\lambda}T_{x=l}) & -\lambda EIV_{x=l} & 0 \\ -(\lambda^3 EIT_{x=l}-C_1 S_{x=l}) & -\lambda^2 EIU_{x=l} & C_2 \end{vmatrix}=0 \qquad (13)$$

The determinant (13) is a frequency equation from which the frequencies of natural vibrations are determined:

$$F(\lambda)=(S_{x=l}-\frac{C_1}{\lambda^3 EI}T_{x=l}-1)(-\lambda EIV_{x=l}C_2)-\frac{1}{\lambda}T_{x=l}(-\lambda^2 EIU_{x=l}+\frac{C_1}{\lambda}T_{x=l})C_2+$$
$$(-\lambda^2 EIU_{x=l}+\frac{C_1}{\lambda}T_{x=l})\lambda^2 EIU_{x=l}+\lambda EIV_{x=l}(\lambda^3 EIT_{x=l}+C_1 S_{x=l})=0 \qquad (14)$$

Equation (14) can be written as:

$$-\lambda^7(EI^2)[(U_{x=l}{}^2)+T_{x=l}V_{x=l}]+\lambda^4 EI[(-C_2 V_{x=l}S_{x=l}+V_{x=l}C_2+C_2\,T_{x=l}U_{x=l}+C_1 T_{x=l}U_{x=l}+C_1 V_{x=l}S_{x=l}]+$$
$$+\lambda C_1 C_2[V_{x=l}T_{x=l}-(T_{x=l}{}^2)]=0$$

The parameter λ is also included in the arguments of circular functions. Assuming one of the supports or 2 supports is rigid, we can consider 3 special cases:

1. $C_1=\infty$

$$F(\lambda)=(S_{x=l})(-\lambda EIV_{x=l}C_2)-\frac{1}{\lambda}T_{x=l}(-\lambda^2 EIU_{x=l})C_2+$$
$$+(-\lambda^2 EIU_{x=l})\lambda^2 EIU_{x=l}+\lambda EIV_{x=l}(-\lambda^3 EIT_{x=l})=0 \qquad (15)$$

2. $C_2=\infty$

$$F(\lambda)=(-\lambda EIU_{x=l}+\frac{C_1}{\lambda}T_{x=l})\lambda^2 EIU_{x=l}+\lambda EIV_{x=l}(-\lambda^3 EIT_{x=l}+C_1 S_{x=l})=0 \qquad (16)$$

3. $C_1=C_2=\infty$

$$F(\lambda)=(-\lambda^2 EIU_{x=l})\lambda^2 EIU_{x=l}+\lambda EIV_{x=l}(-\lambda^3 EIT_{x=l})=0 \qquad (17)$$

Equation (17) corresponds to rigid constraints. The frequency of natural vibrations is expressed in terms of the parameter λ:

$$\omega_k=l^2\lambda_k\sqrt{\frac{EI}{m}} \qquad (18)$$

Knowing the natural frequencies, we can use formula (12) to determine deflections, rotation angles, bending moments, and transverse forces.

With rigid supports $C_1 = C_2 = \infty$, then the frequencies are determined depending on the number of half-waves k by the following formulas:

$$\omega_1 = \frac{\pi^2}{l^2}\sqrt{\frac{EI}{m}}, \qquad k = 1, \qquad \omega_2 = \frac{4\pi^2}{l^2}\sqrt{\frac{EI}{m}}, \qquad k = 2, \qquad \omega_3 = \frac{9\pi^2}{l^2}\sqrt{\frac{EI}{m}}, \qquad k = 3$$

In general, equation (14) can be represented as a nonlinear function of the parameter λ:

$$F(\lambda) = 0 \tag{19}$$

This equation is a transcendental equation with respect to the unknown λ, which is a parameter of the natural frequency ω **(Bekshayev S.Ya., 2013; Sedighi H.M, 2012; Sedighi H.M, 2011; Ruabukhin A.K.,2020)**. This equation has countless solutions, so in the future it is necessary to determine the minimum values of the unknown quantity. Knowing λ, we can find the values of the eigenfrequencies. To solve equation (19), we can use the approximation method. Let's say that two parameter values are found $\lambda = a,b$, where the function $F(\lambda)$ takes two values of different signs, i.e. $F(a)F(b) < 0$. In this case there is at least one point between a and b where $F(\lambda) = 0$. As an initial approximation, we can take the midpoint of the segment $[a,b]$, i.e. $x_0 = \frac{a+b}{2}$.

The iterative process consists of successive refinement of the initial approximation $\lambda = x_0$. Each such step is called an iteration. As a result of iterations, a sequence of approximate values of the root $x_1, x_2, x_3 ... x_n$ is found . If these values approach the true value of the root as n increases, then the iterative process converges. The method of dividing a segment in half is as follows. Let's assume that we found the segment [a,b] that contains the desired root value $x = c$, i.e. $a < c < b$. As an initial approximation of the root with$_0$, we take the midpoint of the segment, i.e. $c_0 = \frac{a+b}{2}$. Next, we study the values of the function $F(x)$ at the ends of the segment $[a, c_0]$ and $[c_0, b]$, i.e. at points a, c_0, b. The one where the function $F(x)$ at the ends of which takes different values contains the desired root, so we take it as a new segment. Discard the second half of the segment $[a,b]$ where $F(x)$ does not change. As the first iteration of the root, we take the middle of the new segment, etc. Thus, after each iteration, the segment on which the root is located is halved, i.e. after n iterations, it is halved.

Let for definiteness $F(a) < 0$, $F(b) > 0$ (Fig. 2). As an initial approximation of the root, we take $c_0 = \frac{a+b}{2}$. Since in the case $F(c_0) < 0 < F(c)$, under consideration, then $c_0 < c < b$ we consider only the segment $[c_0, b]$. Next approximation: $c_1 = \frac{c_0 + b}{2}$. In this case, we discard the segment [c$_1$,b], since $F(c_1) > 0$ and $F(b) > 0$, i.e. $c_0 < c < c_1$. Similarly, we find other approximations: i.e. $c_2 = \frac{c_0 + c_1}{2}$, etc.

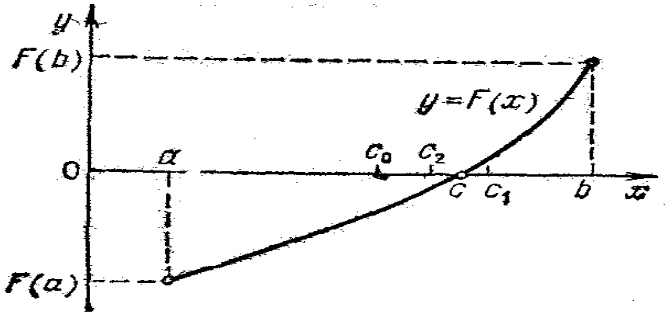

Figure 2 – Iteration method (authors' material)

The iterative process is continued until the value of the function $F(x)$ after n-th iteration becomes less than absolute value of some given value ε, i.e. $|F(c_n)| < \varepsilon$. You can also estimate the length of the resulting segment: if it becomes less than the permissible error, then the calculation stops. **Figure 3** shows a flowchart of this algorithm.

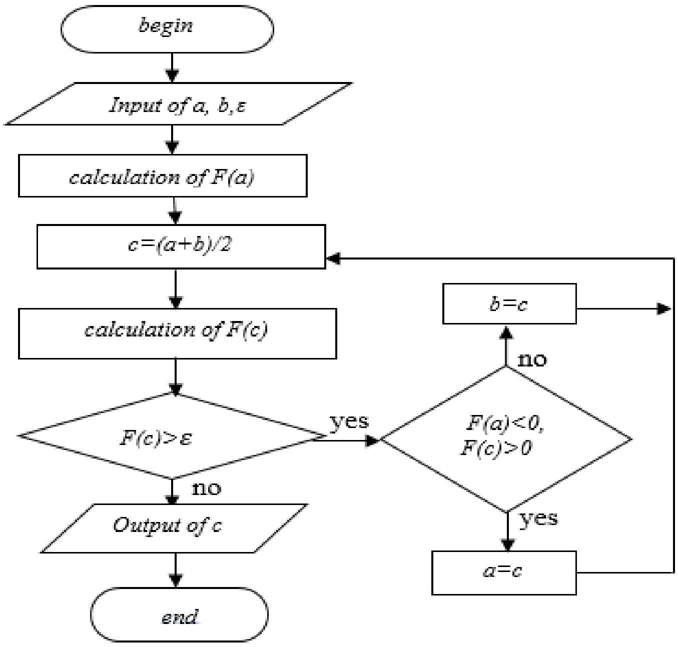

Figure 3 – Flowchart for solving the transcendental equation (authors' material)

By analogy, we can consider the case when the left support of the rod is an absolutely rigid connection, and the right support is elastically pliable (**Figure 4**) **(Ray W. 1977; Korenev B.G., 1972; Nikolaenko N.A., 1988)**.

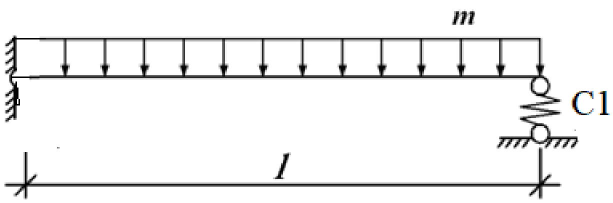

Figure 4 – Design scheme with one elastic support (authors' material)

For the case of a rigid connection at the left end and a hinged support of the beam on the right, the boundary conditions will be:

$$\text{at } x=0 \qquad y_0=0, \quad \varphi_0=0$$
$$\text{at } x=l \qquad y_l=0, \quad Q_l=-y_l\cdot C_1, \tag{20}$$

where y_0 – displacement at the origin, y_l - beam displacement at $x=l$; M_0 and M_l -moment at $x=0$ and $x=l$, respectively, Q_0 and Q_l -transverse forces at $x=0$ and $x=l$, respectively.

Let us express the coefficients A, B, C, and D of solution (7) in terms of the initial parameters y_0 , φ_0, M_0 , Q_0, , then the standing waves can be represented as:

$$\left.\begin{aligned} y_x &= -\frac{M_0}{\lambda^2 EI}U_x - \frac{Q_0}{\lambda^3 EI}V_x \\ \varphi_x &= -\frac{M_0}{\lambda EI} - \frac{Q_0}{\lambda^2 EI}U_x, \\ M_x &= M_0 S_x + \frac{Q_0}{\lambda}T_x \\ Q_x &= M_0\lambda V_x + Q_0 S_x \end{aligned}\right\} \tag{21}$$

In (21) S_x, T_x, U_x, V_x are the Krylov functions:

$$S_x=\frac{ch\lambda x+\cos\lambda x}{2}, \qquad T_x=\frac{sh\lambda x+\sin\lambda x}{2},$$
$$U_x=\frac{ch\lambda x-\cos\lambda x}{2}, \qquad V_x=\frac{sh\lambda x-\sin\lambda x}{2}$$

Using the boundary conditions, we obtain:

$$\begin{aligned} y_{x=0} &= 0 = -\frac{M_0}{\lambda^2 EI}U_{x=0} - \frac{Q_0}{\lambda^3 EI}V_{x=0}, \\ M_{x=l} &= 0 = M_0 S_{x=l} + \frac{Q_0}{\lambda}T_{x=l}, \\ y_{x=l} &= 0 = -\frac{M_0}{\lambda^2 EI}U_{x=l} - \frac{Q_0}{\lambda^3 EI}V_{x=l}, \\ Q_{x=l} &= -y_l\cdot C_1 = M_0\lambda V_{x=l} + Q_0 S_{x=l} \end{aligned} \tag{22}$$

As a result, we obtain a system of three homogeneous equations with respect to three unknowns: M_0, Q_0, y_l. Equating the determinant for the unknowns to zero, we obtain a transcendental equation with respect to the unknown parameter λ.

$$S_{x=l}\left(-\frac{1}{\lambda^3 EI}V_{x=l}C_1 + S_{x=l}\right) - \frac{1}{\lambda}T_{x=l}\left(-\frac{1}{\lambda^2 EI}U_{x=l}C_1 + \lambda V_{x=l}\right) = 0 \tag{24}$$

$$U_{x=0}=\frac{ch0-\cos 0}{2}=0, \qquad V_x=\frac{sh0-\sin 0}{2}=0.$$
$$U_{x=l}=\frac{ch\lambda l-\cos\lambda l}{2}, \qquad V_x=\frac{sh\lambda l-\sin\lambda l}{2}$$

Assuming the right support is rigid, we can consider a special case:

1. $C_1 = \infty$

$$F(\lambda)= S_{x=l}(S_{x=l}) - \frac{1}{\lambda} T_{x=l}(\lambda V_{x=l}) = 0 \qquad (25)$$

4 RESULTS AND DISCUSSIONS

4.1 Results of dynamic calculation of a road overpass

Figure 5 shows a general view of the overpass. The overpass in the longitudinal direction is made three-span. Static system of superstructures is beam-split. Support of beams on supports is hinged.

Superstructure-reinforced concrete, three-span, superstructure No. 1 and No. 3 consists of T-shaped reinforced concrete beams with a length of 11.36 m, made according to the standard design of reinforced concrete prefabricated superstructures without diaphragms with frame reinforcement of a periodic profile.

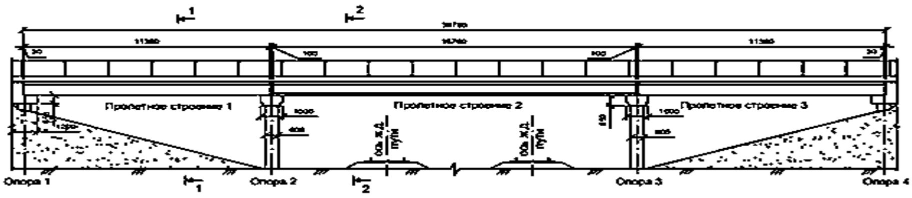

Figure 5 – General view of the overpass (authors' material)

Using the proposed algorithm, the eigenfrequencies of transverse vibrations of spans are determined for the case of hinged fastening of the ends of load-bearing beams $(C_1 = C_2 = \infty)$ and for the case of elastic-yielding supports with different stiffness values (table 1) **(Dostanova S.Kh., 2020; 2021; 2023)**.

Table 1.
Values of natural frequencies and oscillation periods (authors' material)

Span No.	Anchoring conditions	Eigenfrequencies ω_1, Hz	Periods of oscillation sec
1,3	Hinge support $C_1 = C_2 = \infty$	9,82	0,64
2	Hinge support $C_1 = C_2 = \infty$	5,86	1,07
1,3	Elastic support (C_1=C_2=100 kN/cm)	5,52	1,14
2	Elastic support (C_1=C_2=100 kN/cm)	4,84	1,55
1,3	Elastic support (C_1=C_2=200 kN/cm)	5,44	1,15
2	Elastic support (C_1=C_2=200 kN/cm)	4,44	1,25
1,3	Elastic support (C_1=C_2=400 kN/cm)	7,66	0,82
2	Elastic support (C_1=C_2=400 kN/cm)	5,32	1,18
1,3	Elastic support (C_1=C_2=800 kN/cm)	8,55	0,74
2	Elastic support (C_1=C_2=800 kN/cm)	5,38	1,17

Figure 6 shows a graph of changes in the frequency of natural vibrations ω in Hz depending on the stiffness of elastic supports C_1 in kN/cm. For two spans of an automobile overpass. The graph shows 2 asymptotes (1) and (3) corresponding to rigid supports: 1 for the first and third spans; 3 for the second span. Curve 2 corresponds to the first span, and curve 4 corresponds to the second span. It can be seen from the graphs that as the stiffness of the supports increases, the natural oscillation frequencies approach values close to those of rigid supports.

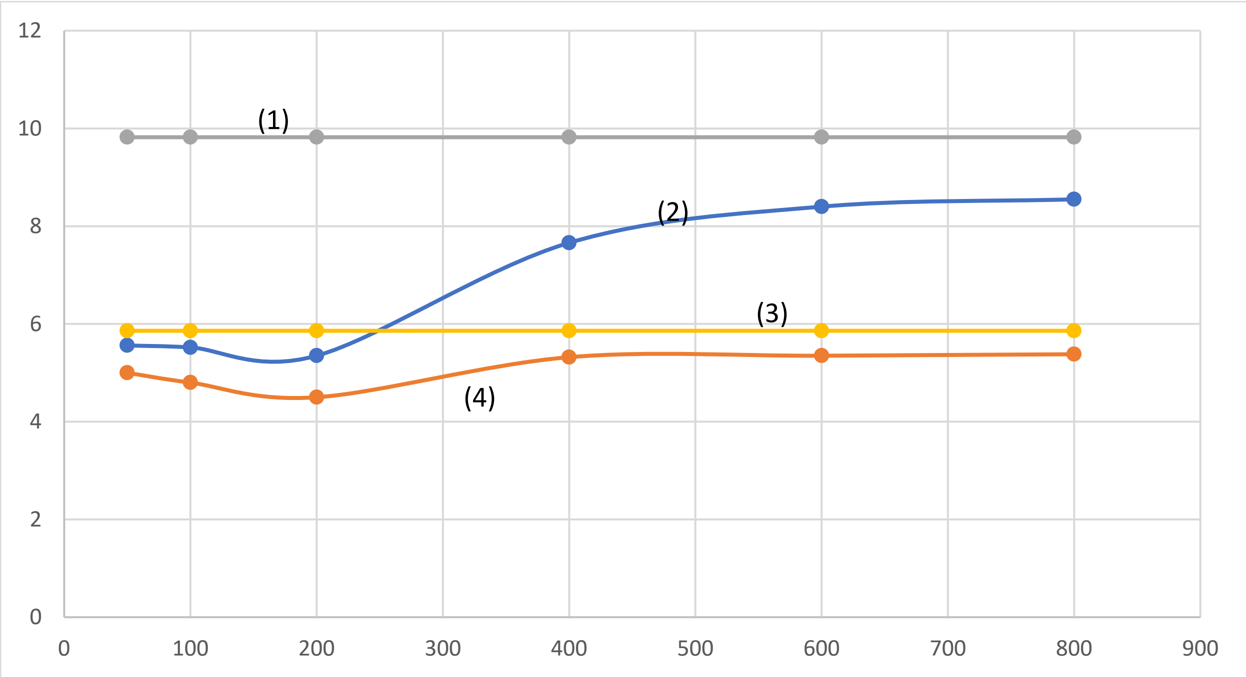

Figure 6 – Graph of changes in the frequency of natural vibrations ω in Hz depending on the stiffness of elastic supports C_1 in kN / cm for two spans of an automobile overpass: (1) for the first, (3) for the second span with rigid supports, (2) corresponds to the first and (4) to the second span for elastic supports (authors' material)

The obtained graphs allow us to evaluate the influence of the stiffness of the supports on the dynamic safety margin of the whole system **(Dostanova S.Kh.2020; 2021; 2023)**. When transport moves at speed v, the dynamic coefficient μ for a single-mass system can be approximated by the following formula:

$$\mu = \frac{1}{1 - \frac{vl}{\pi}\sqrt{\frac{m}{EI}}} = \frac{1}{1 - \frac{v}{\omega} \cdot \frac{\pi}{l}}$$

In this formula m is the span mass, EI is the bending stiffness, l is the span length, ω is the natural frequency. From the obtained graphs it is seen that the most optimal for the 1st and 2nd spans is the stiffness of support links equal to 200 kN / cm. Reduction of frequencies leads to reduction of the value of dynamic coefficient at speed v=40 km/hour by 15-20%. This makes it possible to increase the reserve of dynamic strength by adjusting the stiffnesses of the support links.

5 CONCLUSIONS

The presented algorithm for calculating the dynamic characteristics of transport structures using elastic-yielding supports and the results of theoretical calculations allow us to draw the following conclusions:

1. When using resilient supports in transport structures, you can change the dynamic characteristics and thereby increase the strength reserve and reduce the dynamic effect.

2. Regulating support devices reduce the natural vibration frequencies of the beam, thereby removing it from the resonant zone by 9% or more. By changing the stiffness of the supports, you can optimize the values of dynamic characteristics using the method of successive approximations.

3. An increase in the length of the span beams leads to a decrease in natural frequencies and an increase in internal forces and deflections with free vibrations up to an average of 32%.

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