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RESEARCH ARTICLE

THE IMPACT OF TUNNEL LINING ON THE REACTION OF THE GROUND SURFACE UNDER THE INFLUENCE OF TRANSPORT LOADS

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Abstract. *When a tunnel is subjected to transport loads (loads from the moving transportation within it), vibrations occur in the tunnel lining and the surrounding massif. Traditional quasi-static methods do not account for the dynamic behavior of tunnel structures. Therefore, this paper aims to develop a dynamic calculation method using modern mechanics. The purpose of this paper is to develop such a method. The relevance of the research in this article is due to the trend of increasing the speed of vehicles in recent years. This paper considers an unsupported and lined circular cylindrical shallow tunnel. The tunnel is modeled as an extended circular cylindrical cavity or reinforcing shell located in an elastic half-space. The surface of the cavity or the inner surface of the shell is subjected to a normal load (the effect of the pressure of a moving object on the tunnel) and a tangential load parallel to this axis (the effect of the friction forces of a moving object on the tunnel) moving uniformly along its axis. The motion of the half-space and the shell are described by the dynamic equations of elasticity theory and the equations of classical shell theory, respectively, in moving coordinate systems. The integral Fourier transform method is used to solve the problem. In the case of moving axisymmetric normal and axial tangential loads acting on the tunnel, a numerical study of the influence of the tunnel lining on the stress-strain state of the ground surface is carried out.*

Keywords: *tunnel, elastic half-space, cylindrical shell, transport load, displacements, stresses.*

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ҒЫЛЫМИ МАҚАЛАСЫ

ТОННЕЛЬ ҚАПТАМАСЫНЫҢ КӨЛІК ЖҮКТЕМЕЛЕРІ КЕЗІНДЕ ЖЕР БЕТІНІҢ РЕАКЦИЯСЫНА ӘСЕРІ

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Аңдатпа. *Тоннельге көлік жүктемелері (тоннельде қозғалатын көліктің немесе өзге де объектінің жүктемелері) әсер еткен кезде оның қаптамасының және қоршаған массивтің дірілдері пайда болады. Тоннель құрылымдарының көлік жүктемелерін есептеу үшін қолданылатын шамамен квазистатикалық әдіс олардың динамикалық күй-өзгерістің ерекшеліктерін ескермейді. Сондықтан, механиканың заманауи көріністерін қолдана отырып, математикалық модельдерге негізделген осы конструкцияларды динамикалық есептеудің барабар әдістері қажет. Ұсынылған мақаланың мақсаты - осы әдісті әзірлеу болып табылады. Бұл мақалада қаптамамен бекітілмеген және бекітілген дөңгелек цилиндрлік тоннель қарастырылады. Тоннель серпімді жартылай кеңістікте орналасқан ұзартылған дөңгелек цилиндрлік қуыс немесе оны күшейтетін қабық түрінде модельденеді. Қуыстың бетіне немесе қабықтың ішкі бетіне оның осі бойымен біркелкі қозғалатын қалыпты жүктеме (қозғалатын заттан қысым туннеліне әсер ету) және осы оське параллельді жанама жүктеме (қозғалатын заттан үйкеліс күштерінің туннеліне әсер ету) әсер етеді. Жартылай кеңістік пен қабықтың қозғалысы, сәйкесінше, серпімділік теориясының динамикалық теңдеулерімен және жылжымалы координат жүйелеріндегі классикалық қабық теориясының теңдеулерімен сипатталады. Мәселені шешу үшін интегралды Фурье түрлендіру әдісі қолданылады. Қалыпты және осьтік тангенс жүктемелерінің қозғалмалы осьтік симметриялы тоннельге әсер еткен жағдайда, тоннель қаптамасының жер бетінің кернеулі деформацияланған күйіне әсері сандық зерттеу жүргізілді. Есептеу нәтижелерін талдаудан тоннельді қаптамамен нығайту көлік жүктемелерінің жер бетіне динамикалық әсерінің айтарлықтай төмендеуіне әкелетіні шығарылды.*

Түйін сөздер: *тоннель, серпімді жартылай кеңістік, цилиндрлік қабық, тасымалдау жүктемесі, қозғалыстар, кернеулер.*

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НАУЧНАЯ СТАТЬЯ

ВЛИЯНИЕ ОБДЕЛКИ ТОННЕЛЯ НА РЕАКЦИЮ ЗЕМНОЙ ПОВЕРХНОСТИ ПРИ ТРАНСПОРТНЫХ НАГРУЗКАХ

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Аннотация. *При воздействии на тоннель транспортных нагрузок (нагрузок от движущегося в тоннеле транспорта или иного объекта) возникают вибрации его обделки и окружающего массива. Используемый для расчетов на транспортные нагрузки конструкций тоннелей приближенный квазистатический метод не учитывает особенности их динамического поведения. Поэтому необходимы адекватные методы динамических расчётов данных конструкций, основанные на математических моделях с использованием современных представлений механики. Целью представленной статьи является разработка одного из таких методов. В данной статье рассматривается неподкрепленный и подкрепленный обделкой круговой цилиндрический тоннель мелкого заложения. Тоннель моделируется в виде расположенной в упругом полупространстве протяженной круговой цилиндрической полости или подкрепляющей ее оболочки. На поверхность полости или на внутреннюю поверхность оболочки действуют равномерно движущиеся вдоль ее оси нормальная нагрузка (действие на тоннель давления от движущегося объекта) и параллельная этой оси касательная нагрузка (действие на тоннель сил трения от движущегося объекта). Движение полупространства и оболочки описываются соответственно динамическими уравнениями теории упругости и уравнениями классической теории оболочек в подвижных системах координат. Для решения задачи используется метод интегрального преобразования Фурье. В случае действия на тоннель движущихся осесимметричных нормальной и осевой касательной нагрузок проведено численное исследование влияние обделки тоннеля на напряженно-деформированное состояние земной поверхности.*

Ключевые слова:. *тоннель, упругое полупространство, цилиндрическая оболочка, транспортная нагрузка, перемещения, напряжения.*

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CONFLICT OF INTEREST

The authors state that there is no conflict of interest.

АЛҒЫС/ҚАРЖЫЛАНДЫРУ КӨЗІ

Зерттеу жеке қаржыландыру көздерін пайдалана отырып жүргізілді.

МҮДДЕЛЕР ҚАҚТЫҒЫСЫ

Авторлар мүдделер қақтығысы жоқ деп мәлімдейді.

БЛАГОДАРНОСТИ/ИСТОЧНИК ФИНАНСИРОВАНИЯ

Исследование проводилось с использованием частных источников финансирования.

КОНФЛИКТ ИНТЕРЕСОВ

Авторы заявляют, что конфликта интересов нет.

1 INTRODUCTION

To date, many complex projects of transport tunnels have been created and implemented, and various analytical and numerical methods have been developed to calculate their structures for various types of loads and impacts (**Hrapov et al., 1989**). All this is based on the currently existing design standards for underground structures, which practically do not take into account the speed of transportation or other objects moving in the tunnel, and poorly consider the interaction of its lining with the surrounding massif. Therefore, the calculation of tunnel structures on this basis is very approximate. Since, as it is known from the practice of tunnel operation, at high speed of transport load there is a significant increase of vibration of their structures, adequate methods of dynamic calculation of these structures for transport load, based on mathematical models using modern concepts of mechanics, are required. This paper is devoted to the development of such a method.

When a tunnel supported by a homogeneous cylindrical lining is dynamically designed for transport loads, its design scheme is usually represented as an extended cylindrical shell in an elastic medium. A load (transport load) moving along its axis acts on the inner surface of the shell. The shell is considered in an unbounded medium (elastic space) if the tunnel is deep. However, if it is shallow, it is considered in a medium bounded by a plane parallel to the axis of the shell (elastic half-space).

The problem of the action of a moving normal load on a thin-walled cylindrical shell in elastic space (a model problem for a deeply buried tunnel) was solved in (**Pozhuev & Lvovskij, 1976**). A similar model problem for a shallow buried tunnel is considered in (**Ukrainets, 2006**). Here, a comparative analysis of the stress-strain state (SSS) of the rock mass in the vicinity of an unsupported and supported by a cylindrical lining shallow tunnel under the action of normal transport load is carried out. Due to the fact that the vehicle (or other object) moving along the tunnel, which transfers the normal compressive load to its surface, can have a significant influence on it by friction in the axial direction, (for example, when the wheels of a rolling stock car jam), it is necessary to perform a similar to (**Ukrainets, 2006**) study in the case of their joint action. The results of such a study are presented in this paper.

2 LITERATURE REVIEW

Many works are devoted to the study of the dynamics of extended underground structures under the action of various disturbances. A rather detailed bibliography on this subject can be found in the monographs of Zh.S. Erzhanov, Sh.M. Ajtaliyev, (**Yerzhanov & Ajtaliev, 1989**), Sh.M. Ajtaliyev, L.A. Alekseeva, Sh.M. Dildabayev, N.B. Zhanbyrbayev (**Ajtaliev et al., 1992**) and in the review article of Sh.M. Ajtaliyev (**Ajtaliev, 2004**).

The spatial problems of radiation and reflection of elastic waves during the motion of pulsating loads along a tunnel laid in the ground were considered by M.A. Dashevsky (**Dashevskij, 1971a**), (**Dashevskij, 1971b**). Here, a beam of annular non-deformable cross-section located in elastic space was taken as the design scheme of the tunnel lining. The solution of the problems was constructed in the form of series for scalar and vector potentials. Subsequent articles of M.A. Dashevsky (**Dashevskij, 1974**), (**Dashevskij, 1982**) are devoted to the question of determining the level of ground vibrations in the vicinity of the subway track. In (**Dashevskij, 1974**), a plane problem of elasticity theory was investigated for a half-plane with a hole. A more precise approach is proposed in (**Dashevskij, 1982**). Here, the problem of the response of an elastic half-space containing a cavity supported by a cylindrical shell to a pulsating load moving along the shell axis was solved using the reflected source method. Since the method does not allow satisfying the boundary conditions on the free surface of the half-space, the solution of the problem about the normal load moving along the surface of the half-space is used to refine the solution. The paper proposes an iterative process using these two solutions to obtain the exact solution.

An exact solution of the problem of elasticity theory about an axisymmetric normal load moving along the inner surface of a homogeneous shell located in a boundless elastic medium with a constant subsonic velocity (lower than the shear wave propagation velocities in the shell and the medium) was obtained by V.M. Lvovsky, V.I. Onishchenko and V.I. Pozhuyev (**Lvovsky et al., 1974**). Here, the motions of the shell and the medium were described by the dynamic equations of elasticity theory, and the Fourier integral transformation on the axial moving coordinate was used in the solution. The contact between the shell and the medium was assumed to be sliding. The solution is obtained when the load velocity is less than its critical velocity. In a simplified formulation, when the shell motion is described by approximate equations of the shell theory (classical and Timoshenko type), the solution of the problem (**Lvovsky et al., 1974**) was obtained by V.I. Pozhuyev and V.M. Lvovsky (**Pozhuev & Lvovskij, 1976**). It was found that if the ratio of the thickness of the shell to the radius of its median surface is less than 0.05, then in this case we can use the classical shell theory as the simplest one. It was found that if the ratio of the thickness of the shell to the radius of its median surface is less than 0.05, then in this case the classical shell theory can be used as the simplest one. This position was reflected in further studies of V.I. Pozhuyev (**Pozhuev, 1978**), (**Pozhuev, 1980**).

The action of a load arbitrarily dependent on axial and angular coordinates on the inner surface of a thin-walled shell located in elastic space and moving along its axis at a constant subsonic velocity (less than the velocity of shear wave propagation in elastic space) was considered in (**Ukrainets & Girnis, 2005**), (**Ukrainets & Girnis, 2006**), (**Girnis, 2009**). Here, the motion of the elastic space was described by the dynamic equations of elasticity theory in Lamé potentials, and the shell vibrations were described by the classical equations of shell theory. The equations were represented in a moving cylindrical coordinate system that moved with the load. Initially, an arbitrary load moving in the circumferential direction was assumed to be sinusoidal along the shell axis. The method of incomplete separation of variables was used to solve this problem. The Lamé potentials were represented as the Fourier-Bessel series. The unknown coefficients were determined from the boundary conditions. The obtained solution was then used to solve the problem of the action of a moving load on the given shell, which has no periodicity but is represented as a Fourier integral. As a result, a steady-state solution of the problem was obtained for precritical load velocities. A similar solution of the model problem for a shallow transport tunnel is presented in (**Alekseeva & Ukrainets, 2009**), where the effect of waves reflected from the ground surface, resulting from the movement of the loads on the tunnel lining and the surrounding massif, is additionally taken into account.

In this paper, in contrast to (**Alekseeva & Ukrainets, 2009**), the integral Fourier transform of the axial moving coordinate is used to solve the problem, which allows us to consider the load distributed along the axis of the cavity or the supporting shell according to an arbitrary law and to obtain the final solution expressions without summation at once.

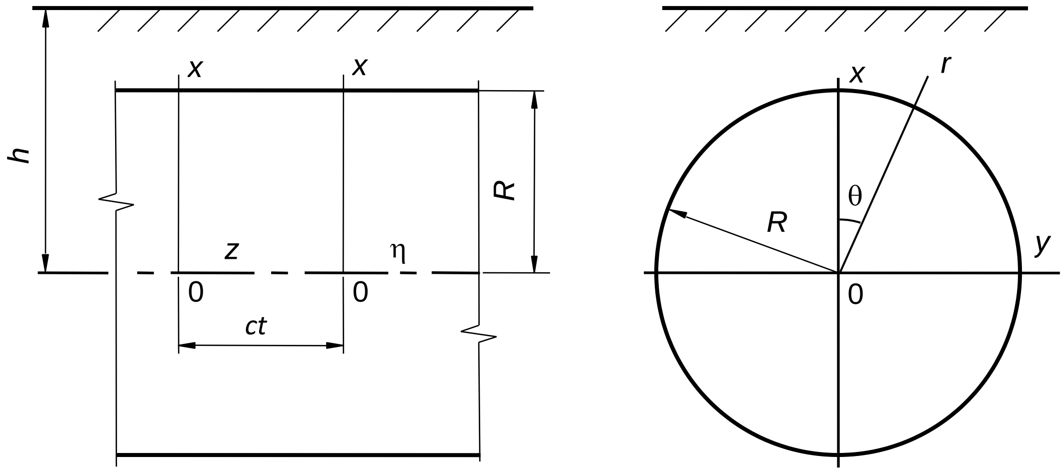
3 MATERIALS AND METHODS

The study uses the method of mathematical modeling with the involvement of models and equations from the theory of elasticity. The design scheme of a shallow transport tunnel is considered as an extended circular cylindrical cavity located in an elastic half-space (for an unsupported tunnel) or a supporting shell (for a tunnel supported by a circular cylindrical lining). The surface of the cavity or the inner surface of the shell is subjected to a normal load (the effect of the pressure of a moving object on the tunnel) and a tangential load parallel to this axis (the effect of the friction forces of a moving object on the tunnel) moving uniformly along its axis. It is assumed that the load functions can be decomposed into a Fourier series in the angular coordinate and a Fourier integral in the axial coordinate. The motion of the shell is described by the classical equations of shell theory, and that of the elastic half-space by the dynamic equations of elasticity in the Lamé potentials, for the solution of which the method of Fourier integral transformation in the axial moving coordinate is used.

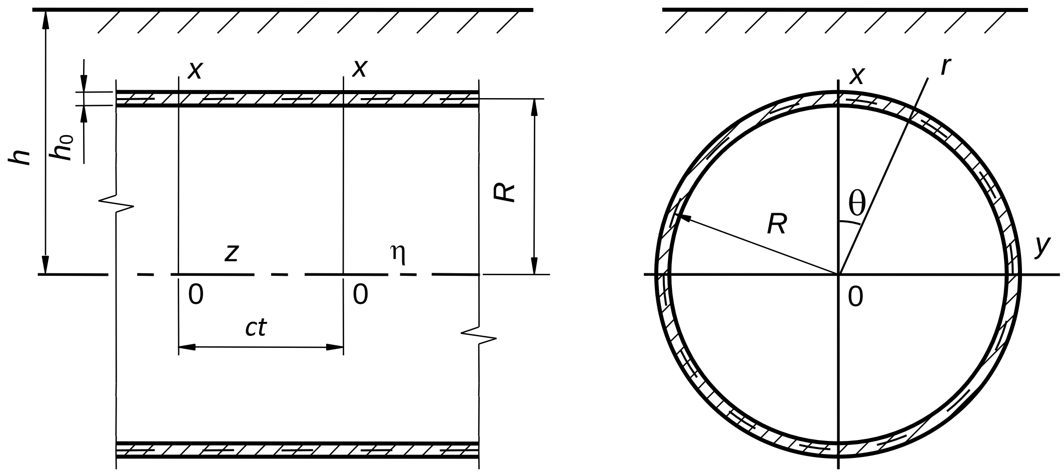
4 RESULTS AND DISCUSSION

4.1 FORMULATION AND ANALYTICAL SOLUTION OF THE PROBLEM

Considering two design schemes for the tunnel (**Figure 1**).



(*a*)



(*b*)

Figure 1 – Design schemes of tunnels: (*a*) unsupported and (*b*) supported by a thin-walled (authors' materials).

In the first design scheme for the tunnel, a linear-elastic, homogeneous, and isotropic half-space (massif) in a cylindrical r, θ, z and Cartesian x, y, z coordinate system that remains unchanged in its position was considered. The half-space, with its horizontal boundary (ground surface) free from loads, contains an extended circular cylindrical cavity with a radius of *R*. The axis of the cavity coincides with the *z*-axis, which is parallel to the boundary of the half-space. The *x*-axis is perpendicular to this boundary of the half-space: $x \le h\,(h > R)$, where *h* represents the depth of the tunnel embedding (**Figure 1 (*a*)**). In the second design scheme of the tunnel, the cavity is fortified with a thin-walled shell (lining) of a thickness, denoted as h_0, and having a radius of the middle surface *R* (**Figure 1 (*b*)**). Considering the thinness of the shell h_0 it is assumed that it contacts with the massif along its middle surface. The shell has hard contact with the massif. The massif and the shell materials are characterized by the following constants, representing their physical and mechanical properties. Poisson's ratio: ν (for the massif), ν_0 (for the shell); shear modulus: μ (for the massif), μ_0 (for the shell); density: ρ (for the massif), ρ_0 (for the shell).

At the surface of the cavity (**Figure 1 (*a*)**) or on the inner surface of the shell (**Figure 1 (*b*)**), there are normal and tangential acting parallel to the *z*-axis loads that move in the direction of the *z*-axis with a constant velocity *c* (lower than the shear wave propagation velocity in the medium). The next stage is to determine the SSS of the massif.

To solve the problem, moving cylindrical $(r, \theta, \eta = z - ct)$ and Cartesian $(x, y, \eta = z - ct)$ coordinate systems that move together with the load are to be introduced. The motion of the shell in these coordinate systems is described by equation (1), while the motion of the massif – by equation (2) (**Ukrainets, 2006**), (**Alekseeva & Ukrainets, 2009**):

$$\left[1-\frac{(1-\nu_0)\rho_0 c^2}{2\mu_0}\right]\frac{\partial^2 u_{0\eta}}{\partial\eta^2}+\frac{1-\nu_0}{2R^2}\frac{\partial^2 u_{0\eta}}{\partial\theta^2}+\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\theta}}{\partial\eta\partial\theta}+\frac{\nu_0}{R}\frac{\partial u_{0r}}{\partial\eta}=\frac{1-\nu_0}{2\mu_0 h_0}\left(P_\eta-q_\eta\right),$$

$$\frac{1+\nu_0}{2R}\frac{\partial^2 u_{0\eta}}{\partial\eta\partial\theta}+\frac{(1-\nu_0)}{2}\left(1-\frac{\rho_0 c^2}{\mu_0}\right)\frac{\partial^2 u_{0\theta}}{\partial\eta^2}+\frac{1}{R^2}\frac{\partial^2 u_{0\theta}}{\partial\theta^2}+\frac{1}{R^2}\frac{\partial u_{0r}}{\partial\theta}=-\frac{1-\nu_0}{2\mu_0 h_0}q_\theta, \quad (1)$$

$$\frac{\nu_0}{R}\frac{\partial u_{0\eta}}{\partial\eta}+\frac{1}{R^2}\frac{\partial u_{0\theta}}{\partial\theta}+\frac{h_0^2}{12}\nabla^2\nabla^2 u_{0r}+\frac{(1-\nu_0)\rho_0 c^2}{2\mu_0}\frac{\partial^2 u_{0r}}{\partial\eta^2}+\frac{u_{0r}}{R^2}=-\frac{1-\nu_0}{2\mu_0 h_0}\left(P_r-q_r\right).$$

Here q_j and u_{0j} – respective the massif reaction and the displacements of points on the middle surface of the shell (when $r = R$ $q_j = \sigma_{rj}$, where σ_{rj} – the stresses in the massif), $j = \eta, \theta, r$; $P_\eta(\theta, \eta)$ and $P_r(\theta, \eta)$ – the intensity of the axial tangential and normal load.

$$\left(M_p^{-2} - M_s^{-2}\right)\text{grad}\,\text{div}\,\mathbf{u} + M_s^{-2}\nabla^2\mathbf{u} = \partial^2\mathbf{u}/\partial\eta^2\,, \quad (2)$$

where $M_p = c/c_p$, $M_s = c/c_s$ – Mach numbers; $c_p = \sqrt{(\lambda + 2\mu)/\rho}$, $c_s = \sqrt{\mu/\rho}$ – the speeds of propagation of compression-expansion and shear waves in the massif, $\lambda = 2\mu\nu/(1-2\nu)$; **u** – vector displacement of the elastic medium, ∇^2 – Laplace operator.

Write **u** through Lamé potentials φ_j ($j = 1, 2, 3$) (**Novackij, 1975**)

$$\mathbf{u} = \text{grad}\,\varphi_1 + \text{rot}\left(\varphi_2 \mathbf{e}_\eta\right) + \text{rot}\,\text{rot}\left(\varphi_3 \mathbf{e}_\eta\right),$$

transform (2) to the form of

$$\nabla^2\varphi_j = M_j^2\,\partial^2\varphi_j/\partial\eta^2\,,\ j = 1, 2, 3. \quad (3)$$

Here $M_1 = M_p, M_2 = M_3 = M_s$.

Let's express the components of the medium's SSS in terms of Lamé potentials φ_j.

Components of the vector **u** in Cartesian (4) and cylindrical (5) coordinate systems:

$$
\begin{aligned}
u_x &= \frac{\partial \varphi_1}{\partial x} + \frac{\partial \varphi_2}{\partial y} + \frac{\partial^2 \varphi_3}{\partial x \partial \eta}, \\
u_y &= \frac{\partial \varphi_1}{\partial y} - \frac{\partial \varphi_2}{\partial x} + \frac{\partial^2 \varphi_3}{\partial y \partial \eta}, \\
u_\eta &= \frac{\partial \varphi_1}{\partial \eta} + m_s^2 \frac{\partial^2 \varphi_3}{\partial \eta^2};
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
u_r &= \frac{\partial \varphi_1}{\partial r} + \frac{1}{r}\frac{\partial \varphi_2}{\partial \theta} + \frac{\partial^2 \varphi_3}{\partial \eta \partial r}, \\
u_\theta &= \frac{1}{r}\frac{\partial \varphi_1}{\partial \theta} - \frac{\partial \varphi_2}{\partial r} + \frac{1}{r}\frac{\partial^2 \varphi_3}{\partial \eta \partial \theta}, \\
u_\eta &= \frac{\partial \varphi_1}{\partial \eta} + m_s^2 \frac{\partial^2 \varphi_3}{\partial \eta^2}.
\end{aligned} \tag{5}
$$

Using Hooke’s law, taking into account (4) and (5), expressions for the components of the stress tensor in Cartesian (6) and cylindrical (7) coordinates can be found

$$
\begin{aligned}
\sigma_{\eta\eta} &= (2\mu + \lambda M_p^2)\frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu m_s^2 \frac{\partial^3 \varphi_3}{\partial \eta^3}, \\
\sigma_{yy} &= \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial y^2} - \frac{\partial^2 \varphi_2}{\partial x \partial y} + \frac{\partial^3 \varphi_3}{\partial y^2 \partial \eta}\right), \\
\sigma_{xx} &= \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial x \partial y} + \frac{\partial^3 \varphi_3}{\partial x^2 \partial \eta}\right), \\
\sigma_{x\eta} &= \mu\left(2\frac{\partial^2 \varphi_1}{\partial \eta \partial x} + \frac{\partial^2 \varphi_2}{\partial y \partial \eta} + (1 + m_s^2)\frac{\partial^3 \varphi_3}{\partial \eta^2 \partial x}\right), \\
\sigma_{\eta y} &= \mu\left(2\frac{\partial^2 \varphi_1}{\partial y \partial \eta} - \frac{\partial^2 \varphi_2}{\partial x \partial \eta} + (1 + m_s^2)\frac{\partial^3 \varphi_3}{\partial y \partial \eta^2}\right), \\
\sigma_{xy} &= 2\mu\left(\frac{\partial^2 \varphi_1}{\partial x \partial y} - \frac{\partial^2 \varphi_2}{\partial x^2} - \frac{m_s^2}{2}\frac{\partial^2 \varphi_2}{\partial \eta^2} + \frac{\partial^3 \varphi_3}{\partial x \partial y \partial \eta}\right);
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
\sigma_{\eta\eta} &= (2\mu + \lambda M_p^2)\frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu m_s^2 \frac{\partial^3 \varphi_3}{\partial \eta^3}, \\
\sigma_{\theta\theta} &= \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + \frac{2\mu}{r}\left(\frac{1}{r}\frac{\partial^2 \varphi_1}{\partial \theta^2} + \frac{\partial \varphi_1}{\partial r} + \frac{1}{r}\frac{\partial \varphi_2}{\partial \theta} - \frac{\partial^2 \varphi_2}{\partial r \partial \theta} + \frac{1}{r}\frac{\partial^3 \varphi_3}{\partial \theta^2 \partial \eta} + \frac{\partial^2 \varphi_3}{\partial r \partial \eta}\right), \\
\sigma_{rr} &= \lambda M_p^2 \frac{\partial^2 \varphi_1}{\partial \eta^2} + 2\mu\left(\frac{\partial^2 \varphi_1}{\partial r^2} + \frac{1}{r}\frac{\partial^2 \varphi_2}{\partial \theta \partial r} - \frac{1}{r^2}\frac{\partial \varphi_2}{\partial \theta} + \frac{\partial^3 \varphi_3}{\partial r^2 \partial \eta}\right), \\
\sigma_{r\eta} &= \mu\left(2\frac{\partial^2 \varphi_1}{\partial \eta \partial r} + \frac{1}{r}\frac{\partial^2 \varphi_2}{\partial \theta \partial \eta} + (1 + m_s^2)\frac{\partial^3 \varphi_3}{\partial \eta^2 \partial r}\right), \\
\sigma_{\eta\theta} &= \mu\left(\frac{2}{r}\frac{\partial^2 \varphi_1}{\partial \theta \partial \eta} - \frac{\partial^2 \varphi_2}{\partial r \partial \eta} + \frac{(1 + m_s^2)}{r}\frac{\partial^3 \varphi_3}{\partial \theta \partial \eta^2}\right),
\end{aligned} \tag{7}
$$

$$\sigma_{r\theta} = 2\mu\left(\frac{1}{r}\frac{\partial^2\varphi_1}{\partial\theta\partial r} - \frac{1}{r^2}\frac{\partial\varphi_1}{\partial\theta} - \frac{\partial^2\varphi_2}{\partial r^2} - \frac{m_s^2}{2}\frac{\partial^2\varphi_2}{\partial\eta^2} + \frac{1}{r}\frac{\partial^3\varphi_3}{\partial r\partial\eta\partial\theta} - \frac{1}{r^2}\frac{\partial^2\varphi_3}{\partial\eta\partial\theta}\right).$$

Applying the Fourier transform in η to equations (3), we get

$$\nabla_2^2\varphi_j^* - m_j^2\xi^2\varphi_j^* = 0, \quad j = 1, 2, 3, \tag{8}$$

where $\varphi_j^*(r,\theta,\xi) = \int_{-\infty}^{\infty}\varphi_j(r,\theta,\eta)e^{-i\xi\eta}d\eta$, $m_j^2 = 1 - M_j^2$, $m_1 \equiv m_p, m_2 = m_3 \equiv m_s$, ∇_2^2 – two-dimensional Laplace operator.

Applying the Fourier transform to (4) – (7) in η, we obtain expressions for the transforms of displacements u_l^* and stresses σ_{lm}^* in Cartesian $(l, m = x, y, \eta)$ and cylindrical $(l, m = r, \theta, \eta)$ coordinate systems, represented in terms of φ_j^*. Applying the Fourier transform to equations (4) – (7) in η, expressions for the transforms of displacements u_l^* and stresses σ_{lm}^* in Cartesian $(l, m = x, y, \eta)$ and cylindrical $(l, m = r, \theta, \eta)$ coordinate systems are obtained, represented in terms of φ_j^*.

If $c < c_s$, then $M_s < 1$ ($m_s > 0$). Therefore, the solutions of equations (8) can be presented in the form of:

$$\varphi_j^* = \Phi_j^{(1)} + \Phi_j^{(2)}, \tag{9}$$

where $\Phi_j^{(1)} = \sum_{n=-\infty}^{\infty} a_{nj}K_n(k_j r)e^{in\theta}$, $\Phi_j^{(2)} = \int_{-\infty}^{\infty} g_j(\xi,\zeta)\exp\left(iy\zeta + (x-h)\sqrt{\zeta^2 + k_j^2}\right)d\zeta$, $K_n(kr)$ – MacDonald functions, $k_j = m_j\xi$; a_{nj}, $g_j(\xi,\zeta)$ – unknown functions and coefficients to be determined, $j = 1, 2, 3$.

The solutions (9) yield the subsequent expressions for φ_j^* in the Cartesian coordinate system:

$$\varphi_j^* = \int_{-\infty}^{\infty}\left[\frac{e^{-xf_j}}{2f_j}\sum_{n=-\infty}^{\infty} a_{nj}\Phi_{nj} + g_j(\xi,\zeta)e^{(x-h)f_j}\right]e^{iy\zeta}d\zeta, \tag{10}$$

where $f_j = \sqrt{\zeta^2 + k_j^2}, \Phi_{nj} = \left(\frac{\zeta + f_j}{k_j}\right)^n$, $j = 1, 2, 3.$

Let's express the functions $g_j(\xi,\zeta)$ using the coefficients a_{nj} ($j = 1, 2, 3$). Considering (10), let's use the boundary conditions when $x = h$:

$$\sigma_{xx}^* = \sigma_{xy}^* = \sigma_{x\eta}^* = 0.$$

Extracting coefficients of $e^{iy\zeta}$ and equating them to zero, due to the arbitrariness of *y*, a system of three equations is derived from which one can deduce

$$g_j(\xi,\zeta) = \frac{1}{\Delta_*}\sum_{l=1}^{3}\Delta_{jl}^* e^{-hf_l}\sum_{n=-\infty}^{\infty} a_{nl}\Phi_{nl}. \tag{11}$$

Here $\Delta_* = \left(2\rho_*^2 - \beta^2\right)^2 - 4\rho_*^2 \sqrt{\rho_*^2 - \alpha^2}\sqrt{\rho_*^2 - \beta^2}$,

$$\Delta_{11}^* = \frac{\Delta_*}{2\sqrt{\rho_*^2 - \alpha^2}} - \frac{\left(2\rho_*^2 - \beta^2\right)^2}{\sqrt{\rho_*^2 - \alpha^2}}, \Delta_{12}^* = -2\zeta\left(2\rho_*^2 - \beta^2\right), \Delta_{13}^* = 2\xi\left(2\rho_*^2 - \beta^2\right)\sqrt{\rho_*^2 - \beta^2},$$

$$\Delta_{21}^* = -\frac{M_s^2}{m_s^2}\Delta_{12}^*, \quad \Delta_{22}^* = -\frac{\Delta_{**}}{2\sqrt{\rho_*^2 - \beta^2}}, \quad \Delta_{23}^* = -4\xi\zeta\frac{M_s^2}{m_s^2}\sqrt{\rho_*^2 - \alpha^2}\sqrt{\rho_*^2 - \beta^2},$$

$$\Delta_{31}^* = -\frac{\Delta_{13}^*}{m_s^2\xi^2}, \quad \Delta_{32}^* = \frac{\Delta_{21}^*}{\beta^2}, \quad \Delta_{33}^* = -\frac{\Delta_{**}}{2\sqrt{\rho_*^2 - \beta^2}} + \frac{\left(2\rho_*^2 - \beta^2\right)^2}{\sqrt{\rho_*^2 - \beta^2}},$$

$$\alpha = M_p\xi, \quad \beta = M_s\xi, \quad \rho_*^2 = \xi^2 + \zeta^2, \quad \Delta_{**} = \left(2\rho_*^2 - \beta^2\right)^2 - 4\rho_{**}^2\sqrt{\rho_*^2 - \alpha^2}\sqrt{\rho_*^2 - \beta^2},$$

$$\rho_{**}^2 = \xi^2 + \left(2/m_s^2 - 1\right)\zeta^2.$$

Note that $\Delta_*(\rho_*)$ – is Rayleigh's determinant, which equals zero when $\rho_{*R}^2 = \xi^2 M_R^2$, or at two points $\pm\zeta_R = \pm|\xi|\sqrt{M_R^2 - 1}$, where $M_R = c/c_R$ – Mach number, c_R – Rayleigh surface wave velocity (**Novackij, 1975**). From the latter, it follows that $\Delta_*(\rho_*)$ does not equal zero on the real axis if $M_R < 1$, or $c < c_R$.

If $c < c_R$ the relations (11), considering (10), will be rewritten as

$$\varphi_j^* = \int_{-\infty}^{\infty}\left[\frac{e^{-xf_j}}{2f_j}\sum_{n=-\infty}^{\infty} a_{nj}\Phi_{nj} + e^{(x-h)f_j}\sum_{l=1}^{3}\frac{\Delta_{jl}^*}{\Delta_*}e^{-hf_l}\sum_{n=-\infty}^{\infty} a_{nl}\Phi_{nl}\right]e^{iy\zeta}d\zeta. \tag{12}$$

Using relation (**Yerzhanov & Ajtaliev, 1989**)

$$\exp\left(iy\zeta + (x-h)\sqrt{\zeta^2 + k_j^2}\right) = \sum_{n=-\infty}^{\infty} I_n(k_j r)e^{in\theta}\left[\left(\zeta + \sqrt{\zeta^2 + k_j^2}\right)/k_j\right]^n e^{-h\sqrt{\zeta^2 + k_j^2}},$$

we obtain (9) in a cylindrical coordinate system (9)

$$\varphi_j^* = \sum_{n=-\infty}^{\infty}\left(a_{nj}K_n(k_j r) + I_n(k_j r)\int_{-\infty}^{\infty} g_j(\xi,\zeta)\,\Phi_{nj}e^{-hf_j}d\zeta\right)e^{in\theta},$$

where $I_n(kr)$ – modified Bessel functions.

If $c < c_R$ the last expression, taking into account (11), can be rewritten as

$$\varphi_j^* = \sum_{n=-\infty}^{\infty}\left(a_{nj}K_n(k_j r) + b_{nj}I_n(k_j r)\right)e^{in\theta}. \tag{13}$$

Here $b_{nj} = \sum_{l=1}^{3}\sum_{m=-\infty}^{\infty} a_{ml}A_{nj}^{ml}$, $A_{nj}^{ml} = \int_{-\infty}^{\infty}\frac{\Delta_{jl}^*}{\Delta_*}\Phi_{ml}\Phi_{nj}e^{-h(f_l+f_j)}d\zeta$.

Using (12) and (13), expressions for the displacement transformant u_l^* and stress transformant σ_{lm}^* are obtained in Cartesian $(l, m = x, y, \eta)$ and cylindrical $(l, m = r, \theta, \eta)$ coordinate systems with unknown coefficients a_{nj} ($j=1,2,3$), which are determined from the boundary conditions on the surface of the cavity $r=R$:

- for a nonreinforced cavity (**Figure 1 (*a*)**)

$$\sigma^*_{rr} = P^*_r(\theta,\xi)\,,\ \sigma^*_{r\theta}=0\,,\ \sigma^*_{r\eta} = -P^*_\eta(\theta,\xi)\,, \tag{14}$$

- for a cavity reinforced by a thin shell (**Figure 1 (*b*)**)

$$u^*_l = u^*_{0l}\,. \tag{15}$$

Here

$$P^*_j(\theta,\xi)=\int_{-\infty}^{\infty} P_j(\theta,\eta)e^{-i\xi\eta}d\eta = p_j(\theta)p^*_j(\xi),\ p_j(\theta)=\sum_{n=-\infty}^{\infty}P_{nj}e^{in\theta},\ p^*_j(\xi)=\int_{-\infty}^{\infty}p_j(\eta)e^{-i\xi\eta}d\eta,\ j=r,\eta;$$

$$u^*_{0l}(\theta,\xi)=\int_{-\infty}^{\infty}u_{0l}(\theta,\eta)e^{-i\xi\eta}d\eta,\ l=\eta,\theta,r\,.$$

By performing the Fourier transform on (1) concerning η and decomposing the functions $P^*_j(\theta,\xi)$ and $u^*_{0l}(\theta,\xi)$ ($j=\eta, r$, $l=\eta,\theta,r$) in the Fourier series in θ, the following is obtained:

$$\begin{aligned}
&\varepsilon_1^2 u_{0n\eta} + \nu_* n\xi_0 u_{0n\theta} - 2i\nu_0\xi_0 u_{0nr} = G_0\left(P_{n\eta}-q_{n\eta}\right),\\
&\nu_* n\xi_0 u_{0n\eta} + \varepsilon_2^2 u_{0n\theta} - 2inu_{0nr} = -G_0 q_{n\theta},\\
&2i\nu_0\xi_0 u_{0n\eta} + 2inu_{0n\theta} + \varepsilon_3^2 u_{0nr} = G_0\left(P_{nr}-q_{nr}\right),
\end{aligned} \tag{16}$$

where $\varepsilon_1^2=\alpha_0^2-\varepsilon_0^2,\ \varepsilon_2^2=\beta_0^2-\varepsilon_0^2,\ \varepsilon_3^2=\gamma_0^2-\varepsilon_0^2,\ \xi_0=\xi R,$

$\alpha_0^2 = 2\xi_0^2+\nu_{**}n^2,\ \beta_0^2=\nu_{**}\xi_0^2+2n^2,\ \gamma_0^2=\chi^2\left(\xi_0^2+n^2\right)^2+2,\ \varepsilon_0^2=\nu_{**}\xi_0^2 M_{s0}^2,$

$\nu_{**}=1-\nu_0,\ \nu_*=1+\nu_0,\ M_{s0}=c/c_{s0},\ c_{s0}=\left(\mu_0/\rho_0\right)^{1/2},\ \chi^2=\dfrac{h_0^2}{6R^2},\ G_0=-\dfrac{\nu_{**}R^2}{\mu_0 h_0};$

P_{nj}, u_{0nl} – the coefficients of decomposition $P^*_j(\theta,\xi)$, $u^*_{0l}(\theta,\xi)$ respectively, in the Fourier series by the angular coordinate θ ($j=\eta, r$, $l=\eta,\theta,r$). When $r=R$ $q_{nl}=(\sigma^*_{rl})_n$ ($l=\eta,\theta,r$).

Resolving (16) concerning u_{0nl} ($l=\eta,\theta,r$), one can find:

$$u_{0n\eta}=G_0\sum_{j=1}^{3}\frac{\delta_{\eta j}}{\delta_n}\left(P_{nj}-q_{nj}\right),$$

$$u_{0n\theta}=G_0\sum_{j=1}^{3}\frac{\delta_{\theta j}}{\delta_n}\left(P_{nj}-q_{nj}\right),$$

$$u_{0nr}=G_0\sum_{j=1}^{3}\frac{\delta_{rj}}{\delta_n}\left(P_{nj}-q_{nj}\right).$$

Here $\delta_n=\delta_{|n|}=\left(\varepsilon_1\varepsilon_2\varepsilon_3\right)^2-\left(\varepsilon_1\xi_1\right)^2-\left(\varepsilon_2\xi_2\right)^2-\left(\varepsilon_3\xi_3\right)^2+2\xi_1\xi_2\xi_3,$

$\delta_{\eta1}=\left(\varepsilon_2\varepsilon_3\right)^2-\xi_1^2,\quad \delta_{\eta2}=\xi_1\xi_2-\xi_3\varepsilon_3^2,\quad \delta_{\eta3}=i\left(\varepsilon_2^2\xi_2-\xi_1\xi_3\right),$

$\delta_{\theta1}=\delta_{\eta2},\quad \delta_{\theta2}=\left(\varepsilon_1\varepsilon_3\right)^2-\xi_2^2,\quad \delta_{\theta3}=i\left(\varepsilon_1^2\xi_1-\xi_2\xi_3\right),$

$\delta_{r1}=-\delta_{\eta3},\quad \delta_{r2}=-\delta_{\theta3},\quad \delta_{r3}=\left(\varepsilon_1\varepsilon_2\right)^2-\xi_3^2,$

$\xi_1 = 2n$, $\xi_2 = 2\nu_0\xi_0$, $\xi_3 = \nu_*\xi_0 n$; $P_{n1} = P_{n\eta}$, $P_{n2} = P_{n\theta} = 0$, $P_{n3} = P_{nr}$, $q_{n1} = q_{n\eta}$, $q_{n2} = q_{n\theta}$, $q_{n3} = q_{nr}$.

By substituting the appropriate expressions into (14) or (15) and equating the coefficients of the Fourier-Bessel series at $e^{in\theta}$, obtain an infinite system ($n = 0, \pm 1, \pm 2, ...$) linear algebraic equations for determining coefficients a_{nj} ($j = 1, 2, 3$). This system of equations has a unique solution if the corresponding determinant $\Delta_n(\xi, c) \neq 0$ is nonzero for each value of *n*. Research on determinants $\Delta_n(\xi, c)$, has demonstrated that for an unsupported cavity (**Figure 1 (*a*)**) this requirement can be fulfilled by satisfying the condition $c < c_R$. However, for a supported cavity (**Figure 1 (*б*)**), the speed ($c < c_R$) of load movement must be lower than its critical speeds: $c < c_{(n)*}$. The values of the critical speeds $c_{(n)*}$ are determined from the dispersion equations $\Delta_n(\xi, c) = 0$ and may be less than the Rayleigh speed c_R. As studies based on numerical calculations show, the lowest critical speed of the load corresponds to the number *n* = 0 ($\min c_{(n)*} = c_{(0)*}$).

We can compute the displacements u_l and stresses σ_{lm} ($l, m = r, \theta, \eta$) in the massif by determining the coefficients a_{nj} ($j = 1, 2, 3$) and applying the inverse Fourier transform.

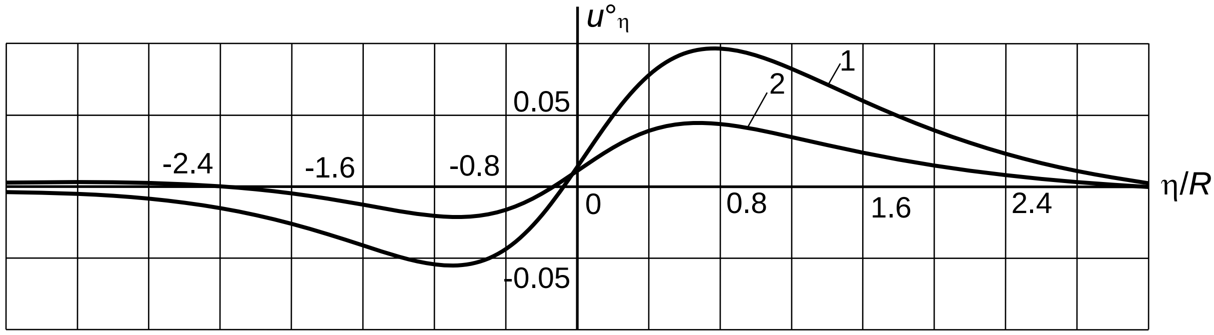
4.2 NUMERICAL EXPERIMENTS

Apply the obtained solution to calculate the SSS of the ground surface. For example, consider an unsupported and supported circular cylindrical tunnel with a thin cast iron lining (h_0 = 0.05 m; ν_0 = 0.3, $\mu_0 = 5.77 \cdot 10^{10}$ Pa, $\rho_0 = 7.2 \cdot 10^3$ kg/m^3), with a radius of *R* = 1 m, passing through siltstone (ν = 0.2, $\mu = 2.532 \cdot 10^9$ Pa, $\rho = 2.5 \cdot 10^3$ kg/m^3, c_s = 1006.4 m/s, c_R = 917 m/s) at a shallow depth *h* = 2*R*.

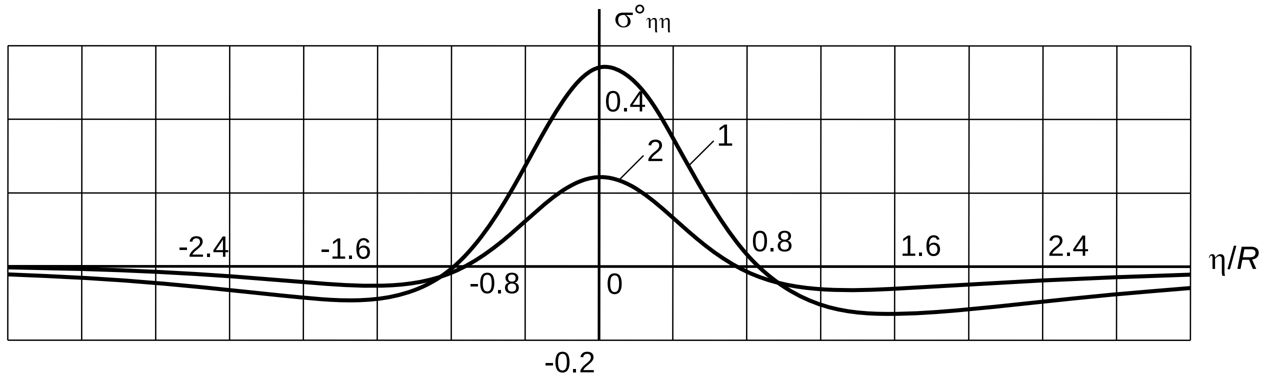
An axially symmetric normal load P_r (pressure of a moving body on the surface of the tunnel) and an axially symmetric tangential load P_η (the result of the friction forces acting on the surface of the tunnel from the moving object), uniformly applied in the interval $|\eta| \leq l_0 = 0.2R$, moving along the tunnel at a constant speed *c* = 100 m/s. In this case $P_{0j} = 1$, $P_{nj} = 0$, $n = \pm 1, \pm 2, ...$, $j = \eta, r$. Let the intensity of the normal load be *Q* (Pa), and the intensity of the tangential load – 0.2*Q*. Then $p_r^*(\xi) = -2Q\sin(\xi l_0)/\xi$, $p_\eta^*(\xi) = 0{,}4Q\sin(\xi l_0)/\xi$. If selected *Q* in such a way that the overall normal load throughout the length $2l_0$ of the load section is equal to an equivalent concentrated radial normal load of intensity $P^{\circ\circ}$ (N/m), i.e. $Q = P^{\circ\circ}/2l_0$, obtain

$$p_r^*(\xi) = -P^{\circ\circ}\sin(\xi l_0)/(\xi l_0),\quad p_\eta^*(\xi) = 0{,}2P^{\circ\circ}\sin(\xi l_0)/(\xi l_0).$$

In **Figure 2** the curves of axial displacements $u_\eta^\circ = u_\eta \mu / P^\circ$ (m) and normal stresses $\sigma_{\eta\eta}^\circ = \sigma_{\eta\eta}/P^\circ$ (where $P^\circ = P^{\circ\circ}$/m, Pa) of the ground surface are shown in the $x\eta$ coordinate plane. The curves labeled 1 represent the unsupported tunnel, while the curves labeled 2 represent the tunnel reinforced with a thin-walled cast iron lining.



(*a*)



(*b*)

Figure 2 – Changes in axial displacements (*a*) and normal stresses (*b*) of the ground surface (authors' materials).

From the analysis of the behaviour of the curves, it follows that in the case of a tunnel reinforced with a cast iron lining extreme axial displacements $u_η$ and normal stresses $σ_{ηη}$ of the ground surface are significantly smaller in absolute magnitude than in the case of an unsupported tunnel. For any of the considered tunnels, when $η \approx 0$ displacement $u_η=0$, while the stresses $σ_{ηη}$ have the maximum value. When $|η|$ increases, $|u_η|$ increase and reach the extreme values when $|η| \approx 0.7R$. Moreover, when $|η| \approx -0.7R$ $u_η < 0$, and when $|η| \approx 0.7R$ $u_η > 0$ and are almost 2 times the value of $|u_η|$, found when $η \approx -0.7R$. With a further increase in $|η|$ there is a damping effect on the displacement of the earth's surface, represented by $|u_η|$. As **Figure 2 (*b*)** shows in the interval $|η| \leq 0.8R$ there is a decrease in tensile stress $σ_{ηη}$ from its maximum value to zero. With increasing in $|η|$, there is an increase in absolute value from zero to certain values of compressive stresses $σ_{ηη}$ (smaller than the maximum stress $σ_{ηη}$ occurring when $η \approx 0$) and their further decay.

The graphs obtained from mentally visualizing the deformation of the ground surface under the influence of these loads support this representation. As the numerical results of the study are unprecedented in the existing literature, no comparison with similar results is made here. The validity of these results is ensured by the correct formulation of the problem, the application of accurate mathematical methods of elasticity theory in its analytical solution, the rigor of the mathematical apparatus used, and the high degree of satisfaction of boundary conditions in the numerical realization of the set problem.

5 CONCLUSIONS

The model problem for an unsupported or supported circular cylindrical tunnel with a thin lining of shallow embedment under the action of transport loads, including normal and tangential loads parallel to the tunnel axis has been solved. This action occurs when taking into account the frictional forces that arise when transport or other objects move through a tunnel.

In contrast to similar model problems for deep-buried transport tunnels, where the massif is typically represented as an elastic space, this model takes into account the impact of waves reflected by the ground surface and arising during the movement of loads on both the tunnel structure and the surrounding massif.

Computer programs developed from the obtained solution were used to conduct a numerical study on the influence of shallow tunnel lining on the SSS of the ground surface. The study involved the application of axisymmetric normal and tangential loads, uniformly distributed within a certain interval and moving at a constant speed. The analysis of the calculation results shows that the reinforcement of the tunnel with the lining leads to a significant reduction of the dynamic impact of the transport loads on the ground surface. The vibration of the ground surface, which can negatively affect the seismic resistance of nearby buildings and structures, depends on the physical and mechanical properties of the material and the thickness of the tunnel lining. Therefore, the choice of material and its thickness can reduce this effect. The obtained solution allows us to study the dynamics of a circular cylindrical tunnel at any depth of its embedment and various permissible speeds of transport loads.

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